Joseph Thwaites 52 Sackville Dublin

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Lord *HARRINGTON.*

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And

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Your Lordship's

most Obedient

Humble Servant,

B. LANGLEY.

CONTENTS.

PART I.

PART II.

PERSPECTIVE made easy.

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Joseph Thwaites 52 Sackville
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THE

YOUNG BUILDER's RUDIMENTS.

PART I.

LECTURE I.

Of the Definitions of such Geometrical Lines, Superfices and solid Bodies, as are necessary to be understood by every Workman concern'd in the noble Art of Sound Building.

A. WHAT do you mean by Definitions?

B. Definitions, are the Explications of such Lines, Figures, Bodies, and Terms, &c. as concern the following Work, towards rendering of it plain, clear, and easy, by which all manner of Difficulties will be avoided.

A. What do you mean by Geometry?

B. Geometry is a part of pure Mathematicks, wherein Magnitude is consider'd, not so much in respect to its self, as the relation it may have to another Magnitude of the same kind, in abstracting it from all Matter or sensible Subject. Wherefore it is divided into two Kinds or Parts: That is, into *Speculative* and Practical.

A. What is Speculative Geometry?

B. Speculative Geometry, considers the Properties of Quantity, which has Extension, as *Lines*, *Planes* and Solids, &c.

A. What is Practical Geometry?

B. Practical Geometry, is the Art of Meaſuring and Dividing of Quantity or Magnitude, as Lines, Superfices, and Solids, *&c.*

A. What is the leaſt Part of Quantity?

B. A Point, as the ſuperficial Appearance made by the Point of a Pen, Pin, Pencil, *&c.* By moſt Mathematicians ſaid to have no Parts, becauſe *Euclid* happen'd to ſay, *That a Point is that which hath no Parts.* That is, a Point is that which is no part of Quantity, but is the beginning and end of Quantity. What a ſtupid Definition is this, when they ſay, *A Point is that which is, &c.* Wherefore it is ſomething, or otherwiſe it could not be the Object of the Mind, and conſequently nothing could be underſtood by it. But if ſomething is to be conceived in the Mind, tho' never ſo infinitely ſmall, it muſt be conſider'd as a ſuperficial Quantity at leaſt, (if not a ſolid,) namely, an infinitely ſmall Sphere, which yet ſo infinitely ſmall may by the Mind (tho' not by the Hand) be again divided and ſub-divided into an infinite Number of leſſer Parts.

Had our *Engliſh* Mathematicians but conſider'd, as Monſieur *Ozanam* did, they would have defin'd a Point as he hath, *viz. A Mathematical Point is that which hath not Parts, or at leaſt is what ought to be conſider'd as ſuch.* He knew well, that a Point did conſiſt of Parts, and therefore ſaid, *It is what is to be conſider'd, or ſuppos'd to have no Parts*; but does not in the leaſt offer, or attempt poſitively, as others have blindly done, to ſay, *That a Point hath no Parts.*

Now, ſince that *Phyſicks* allow a Sub-diviſion of a Point, 'tis ſtrange to me, that Mathematicks, which is the real Subject of Quantity, ſhould deny that a Point is any part of Quantity, and yet is the very firſt Principle from which the whole aſcends.

And in Conſideration that none has yet prov'd a Point to be no Part of Quantity, I ſhall therefore, until ſuch Proof be made publick, define a Point to be the leaſt Magnitude conſider'd, and that two-fold. That is, *Firſt*, The leaſt Superfices when conſider'd as a Surface: And *Secondly*, The leaſt Solid, when conſider'd as a Body. Whence it follows, that Lines will be of two kinds; that is, either ſuperficial when conſider'd on a Plane or Surface, as a Line drawn on Paper with a Pen or Pencil; or ſolid, when abſolute, as a Line ſtrain'd thro' the Air, from one given Point to another.

Since that all Lines in Practice are generated by the Motion of a Point, from one determinate part of Space to another, the ſuperficial Point muſt generate a Superfices, whoſe length is = the Space paſs'd thro',


Plate I
p 3
Fig I
Fig II
Fig III
Fig IIII
Fig V
Fig VI
Fig VII
Fig VIII
Fig IX
Fig X
Fig XI
Fig XII
Fig XIII
Fig XV
Fig XVI
Fig XVII
Fig XVIII
Fig XIX
Fig XX
Fig XXI
Fig XXII
Fig XXIII
Fig XXIIII
Fig XXV
Fig XXVI
Fig XXVII
Fig XXVIII
Fig XXVIIII
Fig XXX
Fig XXXI
Fig XXXII
Fig XXXIII
Fig XXXIIII
Fig XXXV
Fig XXXVI
Fig XXXVII
Fig XXXVIII

thro', and breadth to the Diameter of the Point; and the ſolid Point muſt generate a ſolid Body, whoſe length will be = the Space paſs'd thro', and thickneſs to the thickneſs of the Point.

A. Pray what Forms are theſe two kinds of Points of?

B. The ſuperficial Point is, an infinite ſmall Circle, being the only ſuperficial Figure that contains the moſt Space in the leaſt Dimenſion; wherein obſerve, That if a Point is conſider'd as a Circle, then its Center may in ſome meaſure be conſider'd as a Point. But ſince that that Point doth contain Space, tho' very ſmall, its out Part may be conſider'd as a Circle, and its Center may in like manner be conſider'd as before, and that again in like manner without end.

The ſolid Point is an infinite ſmall Ball, Globe, or Sphere, which contains the moſt Matter in the leaſt Space, and that regular, and at equal diſtances about its Center: Which Center being conſider'd as another Ball, is capable of a Center alſo, and ſo on in like manner may a Point be conſider'd without end.

It is as hard a Task to aſſign the Dimenſions of the leaſt Quantity, as it is to aſſign Bounds to the Univerſe. If this Definition is abſurd I ſhould be very greatly oblig'd to the Perſon that can inform me otherwiſe. Now to the Matter in Hand; wherein I ſhall define all ſuch Lines, Angles, ſuperficial Figures and ſolid Bodies, as are to our Purpoſe.

A. What are the Lines neceſſary to be underſtood?

B. A right Line, and a circular Line.

A. What is a right Line?

B. A right Line is the neareſt Diſtance between two given Points, as the Line or Diſtance between A B. for had the Point A mov'd in the curved Line firſt to C, and afterwards to B, it would have paſs'd thro' a greater Space than the right Line A B. Plate 1. *Fig.* 1.

A. What is a circular Line.

B. A circular Line, is generated by the end of a right Line. Suppoſe the right Line *a d*, fix'd at its end *d* as a Center, then if it be mov'd from *a* to *b*, and from *b* to *c*, its end *a* will deſcribe a curved or circular Line *a b c*, which is alſo call'd, an Arch of a Circle, *Fig.* 2. for was the Point *c* to be moved on to *e*, and from thence to *a*, it would complete a round Space *a b c e a*, which is call'd a Circle; and theſe are the two kinds of Lines that are neceſſary for our following Purpoſes.

A. I thank you *Sir.* Pray what do you mean by an Angle?

 B. An

B. An Angle is an indefinite Space terminated by two right inclining Lines, which meet together in one Point, as the right Lines *d b*,
Fig. 3. and *d e*, which being continued in their own Positions, will meet at *c*, and by that generate an Angle. So likewise, the right Lines *f g*, and *i k*, being continued, will meet at *h*, and form an Angle also. But if two Lines meet in such a manner, as to have no Inclination the one to the other, they will generate a right Line, as the Line *l i*, meeting the Line *m n*, in the Point *o*, and being both on the same Plane or Level, will when they have met, generate a right Line, equal to both their lengths, without forming of any Angle. Therefore if one right Line meet another right Line, in any different Positions, they will constitute an Angle at their Point of meeting.

A. How many kinds of Angles are in Practice.

B. Three. That is to say, an acute Angle, a right Angle, and an obtuse Angle.

A. What is an acute Angle?

B. An acute Angle, is an Angle whose Inclination is nearer than a right Angle; but first I must inform you what a right Angle is; a right Angle is constituted by the meeting of two right Lines, with
Fig. 4. an equal Inclination. That is, if a Line as *e c*, meet another Line as *d b*, and inclines no more towards *d*, then it doth towards *b*, but stands directly square between both, then the Angle is call'd a right Angle, and the Line *e c*, is therefore call'd a perpendicular Line to the Line *d b*.

Now, when any two Lines incline nearer to one another than *d c* doth to *e c*, as the Lines *f c* and *d c*, or *a c* and *d c*, than by their meeting, they form sharper Angles than the right Angle *e c d*, and are therefore all call'd acute Angles.

A. Very well *Sir*, I understand you. But pray what is an obtuse Angle.

B. An obtuse Angle, is an Angle constituted by the meeting of of two right Lines, whose Inclination is greater from one another than the Lines of the right Angle, as the Angles made by the meeting of the Lines *f c*, and *c b*; or *a c*, and *e b*, by which they form Angles that are more blunt than the right Angle, and are therefore call'd obtuse Angles; and these are the several Varieties of Angles.

A. I thank you *Sir*. Pray now proceed to superficial Figures?

B. The superficial Figures for your Purpose, are the Circle, the various kinds of Triangles, the Square, the Parallelogram, the Trapezium, and the various kinds of Polygons.

A. The

A. The Circle you have already ſhewn. Now pray proceed to the Triangles?

B. 'Tis true, I have ſhewn you how a Circle is generated, but I have not inform'd you of the Lines belonging to it: Therefore obſerve, that the curved Line *a b c d e a*, is call'd the Circumference, the Space contain'd within it, the Circle its ſelf. The Point *d* its Center; the Line *a e*, (or any other that paſſes thro' its Center) its Diameter, and one half of it as *a d*, or *d e* is call'd the Semi-diameter, or Radius of the Circle. Now I will proceed to define the various kinds of Triangles.

A. How many kinds of Triangles are there neceſſary to be known?

B. Three. That is to ſay, Equilateral, Iſoſeles, and Scalene.

A. What is an Equilateral Triangle.

B. A Triangle, whoſe three Sides, are = one another, as the Triangle *a*. *Fig.* 5.

A. What is an Iſoſeles Triangle?

B. A Triangle, that hath two Sides = one another, and the third Side unequal, as the Triangle *b*.

A. What is a Scalene Triangle?

B. A Scalene, or Scalenum Triangle, is a Triangle whoſe three Sides are unequal to one another, as the Triangles *c* and *d*, and here Note,

That when one of the Angles of a Scalene Triangle is right angled, as the Triangle *e* right angled at *g*, then ſuch a Triangle is called a right angled plain Triangle, wherein the Side *e g* is called the baſe; the Side *h g* the perpendicular, and the Side *e h*, the Hypothenuſe.

Alſo obſerve, in all Triangles, wherein a Line is drawn from any Angle to the oppoſite Side, and cuts the ſame at right Angles, as the Line *d g* of the Triangle *a*, or the Line *a d*, of the Triangle *b*; or the Line *e a*, of the Triangle *d*; ſuch a Line is always called the perpendicular of the Triangle, and the Side on which it falls, as the Side *n o*, of the Triangle *a*, or the Side *n m*, of the Triangle *d*, is called the Baſe of the Triangle; and theſe are the various kind of plain Triangles.

A. Pray what is a Square?

B. A Geometrical Square is a plain Figure contain'd under four equal right Lines, as *a b c d*, whoſe Angles at *a b c d*, are each right Angles, wherein obſerve, that the Lines *a d*, and *b c*, *Fig.* 6. are

are the Diagonals, the Lines *e h*, and *g f*, the Diameters, and the Point *i*, where they all interfect is the Center.

A. What is a Parallelogram.

B. A Parallelogram is a long Square, having a greater Length than Breadth, wherein it only differs from the Geometrical Square, whofe Length and Breadth are equal, and thereby hath its Diameters un-
Fig. 7. equal as *f c*, the Diameter of its Length, and *b g*, the Diameter of its Breadth: But the Diagonals are equal, and *e*, the Point of their Interfection, is the Center of the Figure.

A. Is there any other Kinds of four-fided Figures?

Yes, there are three others; the Rhombus, the Rhomboides, and the Trapezium.

A. What is a Rhombus?

B. A Rhombus, is a plain Figure, of a Diamond Form, contain'd
Fig. 8. under four equal Sides, as the Figure *c b d e*, whofe oppofite Angles *b* and *e* are obtufe, and *c d*, acute.

A. What is a Rhomboid?

B. A Rhomboid is a Figure of four Sides and Angles oblique, whereof each oppofite Side are parallel to one another, and the op-
Fig. 9. pofite Angles equal to one another, as the *Fig. a b c d.* wherein the Sides *a b*, and *c d* are parallel to each other, as alfo are the Sides *a c* and *b d*, and the Angle *o* is = the Angle *d*, and the Angle *c*, to the Angle *b*.

Wherein 'tis evident, that the Difference between a Rhombus and Rhomboid, is by the Rhomboides having its Length greater than its Breadth, as was before obferv'd in the Geometrical Square, and Parallelogram. And here it is alfo obfervable, that the Difference between a Geometrical Square and a Rhombus, and between a Parallelogram and a Rhomboid, confifts in the Angles only, the Angles in the firft two being all right Angles, and the Angles in the latter two, being all oblique Angles; that is, acute and obtufe, for being either of them, the Angle is call'd an oblique Angle.

A. Very well *Sir*. But you fay that their oppofite Sides are parallel. Pray what is it you mean by parallel Lines.

B. By parallel Lines, you are to underftand, fuch, as being infinitely continu'd either way, would never meet; that is, being always at equal Diftance, without Inclination towards one another, as *a b* and
Fig. 10. *c d*, whofe perpendicular Lines *e f*, *g h i n*, are equal.

A. I underftand you. Pray proceed to the Trapezium.

B. A

B. A Trapezium is a plain Figure, contain'd under four unequal Sides, and as many unequal Angles, as *a b c d*, and thus have I done with the four-ſided Figures. *Fig.* 11.

The next in Order, is the regular Polygons.

A. What is a Polygon?

B. A Polygon is a Figure of more than four Sides, as the Polygons A B C D E F. *Fig.* 12.

A. Are they diſtinguiſh'd by particular Names?

B. Yes, according to the Number of their Sides: Thus, If a Polygon conſiſt of five Sides, as A, 'tis call'd a Pentagon; If of ſix Sides, as B, a Hexagon; If of ſeven Sides, as C, a Heptagon: If of eight Sides, as D, an Octagon; If of nine Sides, as E, a Nonagon; and if of ten Sides, as F, a Decagon; and when all their Sides and Angles are equal, they are call'd regular Polygons.

A. Suppoſe that a right lin'd Figure conſiſts of many Sides, all unequal, as *a b c d e f g h*. What do you call ſuch a Figure? *Fig.* 13.

B. Such a Figure is call'd an irregular Figure; and thus have I done with ſuperficial Figures. The next in Order are the ſolid Bodies.

A. What are they?

B. The Cube, Parallelopipedon, Priſim, Sphere, Cylinder, Cone, Pyramis, and Fruſtums of the Cone or Pyramis.

A. What is a Cube?

B. A Cube is a ſolid Body, conſiſting of three Dimenſions, *viz.* Length, Breadth and Thickneſs (as all other Solids do,) that is, its Length is equal to its Breadth, and its Depth or Thickneſs, to its Length or Breadth.

A Dice is a Cube, all its Sides being equal.

A. Very well *Sir*, I underſtand you that a Cube is a ſolid Body, whoſe Length, Breadth and Depth, are equal to one another, But ſuppoſe that any one of theſe Dimenſions ſhould be unequal; How then pray?

B. Why, when any one of its Dimenſions are unequal, as its Length greater than its Breadth, or its Depth greater or leſs than its Breadth, as the Body B, then it is call'd a Parallelopipedon, or long Cube.

A. I thank you *Sir*, now proceed.

B. The next Solid is the Sphere, Globe, or Ball, which is generated by the Revolution of a Semi-circle about its own Diameter, as

as the Semi-circle *a b e*, being revolved about its own Diameter *a e*, by its Revolution, paſſes through a Space which is equal to a Sphere or Globe of the ſame Diameter.

A. I comprehend you, pray proceed. What is a Cylinder?

B. A Cylinder is a ſolid Body, made by the Revolution of a Parallelogram about one of its Sides, as the Parallelogram *a b c d*, revolving about its Side *d e*, generates the Cylinder D; and ſo in like Manner, the Scalene, or Right-Angled Triangle *a b c*, being revolved about its Side *a c*, generates the Cone *a b c*.

A. And is the Pyrament G generated in the ſame Manner?

B. No; a Pyramid or Pyrament is a Body terminating in a Point as G in the Point *a*, contain'd under four Planes at leaſt, which are Iſoſceles Triangles, the Baſe on which it ſtands, oppoſite to the ſolid Angle or Vertex *a*, excepted, which may be a Triangle, a Square, a Parallelogram, or a Polygon, &c.

N. B. The Line *a e* of the Sphere C, the Line *d c* of the Cylinder D, and the Line *a c* of the Cone F, are called the Axis of each Solid.

Now there only remains the two Fruſtums of the Cone I, and of the Pyramis H; which are no other than the Butt-ends of a Cone or Pyrament, under which Forms round and ſquare Timber commonly happens, when the Tops of the Trees are cut off. Theſe are the ſeveral Definitions neceſſary to be underſtood before you proceed to the next Lecture; and therefore, I adviſe you to go over them with good Attention, at leaſt four times, before you proceed further. After which all the Difficulties will be over, and the enſuing Problems very eaſy and delightful.

LECTURE II.

Of ſuch Geometrical Problems as are neceſſary to be underſtood by every Workman concern'd in the noble Art of ſound Building.

A. WHAT is a Problem?

B. A Problem is a Queſtion which propoſes ſomething to be done, and teaches the Manner how to perform the Operation required, as following.

PROBLEM I.

Fig. 15. To divide the right Line *a c* into two equal Parts by the Perpendicular *a c*.

PRAC-

PRACTICE.

1. Open your Compaſſes to any Diſtance greater than half the given Line *a c*; and on each End, as at *a* and *c*, deſcribe Arches, as *d h* and *f i*, interſecting each other in the Points *e* and *g*. Fig. 15.

2. From the Points *e* and *g* draw the right Line *e g*, which is the Perpendicular required; which will divide *a c* into two equal Parts at the Point *b*.

PROBLEM II.

Fig. 16.

To divide the Angle *b a e* into two equal Parts, by the right Line *a h*.

PRACTICE.

1. On the Point *a*, deſcribe an Arch of a Circle of any Radius, as *c d*, and with the ſame opening of your Compaſſes, on the Points *c* and *d*, deſcribe the Arches *g g* and *f f*, interſecting each other in *h*.

2. From the angular Point *a*, draw to *h* the right Line *a h*, which will divide the Angle into two equal Parts as required.

PROBLEM III.

The right Line *a k* of a certain Length being given, to continue the ſaid Line a longer Length to *e*.

PRACTICE.

1. On *a*, with any opening of your Compaſſes, deſcribe an Arch as *c d*; and from the Point of Interſection *b*, ſet off on the Arch any equal Diſtances to *c* and *d*. Fig. 17.

2. With any large Diſtance greater than *d b*, on the Points *d* and *c* deſcribe Arches, as *f i* and *g h*, interſecting each other in *e*.

3. From the Point *b* to the Point *e*, draw the right Line *b e*, the Continuation required.

PROBLEM IV.

The right Line *g k* being given to draw the right Line *a f*, parallel to it, at the given Diſtance of the Line *l m*. Fig. 18.

PRACTICE.

1. Take the Length of your given Diſtance *l m* in your Compaſſes, and with that opening on any two Points in the Ends of the Line *g k*, as at *h i*, deſcribe Arches, as *b c* and *d e*.

2. Lay a Ruler to the Extreams of those Arches, and draw the right Line *a f*, which will be parallel to *g k*, as required.

PROBLEM V.

Fig. 19. To erect the Perpendicular *b c*, in or near the middle of the right Line *a e*, from the given Point *c*.

PRACTICE.

1. From the given Point *c* set off any equal Distances to *b* and *d*; then with any opening greater than *b c*, on the Points *b* and *d* describe the Arches *f g* and *i k*, intersecting each other in *h*.

Draw the right Line *h c*, and 'tis the Perpendicular required.

PROBLEM VI.

To raise the Perpendicular *c d* from *c*, the End of the right Line *a c*.

PRACTICE.

1. On *c*, with any opening of your Compasses, describe the Arch Fig. 20. *b f g h*, and set the same opening as *c b*, from *b* to *f*, and from *f* to *g*: then with the same, or a greater opening of your Compasses, on the Points *f* and *g*, describe Arches, as *f i* and *g e*, intersecting in *d*.

2. Draw the right Line *d c*, and 'tis the Perpendicular required.

PROBLEM VII.

To let fall the Perpendicular *c d* from the given Point *c*, on the right Line *a b*.

PRACTICE.

1. With any opening of your Compasses greater than *c d*, describe Fig. 21. an Arch as *b e*, intersecting the right Line *a b* in the Points *g* and *n*.

2. With the Distance *g n*, on *g* and *n*, describe the Arches *i i* and *k k*, intersecting each other in *m*; then draw the right Line *c m*, and *c d* will be the Perpendicular let fallen, as required.

PROBLEM VIII.

To make the Angle *f h k* equal to the Angle *c a e*.

PRAC-

PRACTICE.

1. On the given Angle *a*, with any opening of your Compasses, describe an Arch as *b d*, and then having drawn a right Line at Pleasure, as *h k*, on any of its Ends as *h*, with the opening *a d*, describe the Arch *i g*. *Fig.* 23.

2. Make *i n* = *d b*, and then from *h* through *n* draw the right Line *h f*, which completes the Angle *f h k* = the given Angle *c a e*, as required.

PROBLEM IX.

To divide the right Line *a b* into any Number of equal Parts, suppose six.

PRACTICE.

1. From one End of the given Line *a b*, draw another right Line, as *a h* from *a*, making any Angle at Pleasure; then from the other End, as *b*, draw the right Line *b i*, parallel to it, by *Prob.* 4. or make the Angle *a b i* = the Angle *h a b*, by *Prob.* 8.

2. Open your Compasses to any Distance, suppose *a p*, and as the Line is to be divided into six Parts; therefore set off five of those Distances on the Line *a h*, at the Points *p q r s h*, as likewise the same on the Line *b i*, at the Points *o n m l k*.

3. Draw the Lines *p k*, *q l*, *r m*, *s n*, and *h o*, and they will divide the given Line *a b* into six equal Parts, at the Points *c d e f g*, as required.

PROBLEM X.

To divide the Circumference of the Circle *c a e d* into 360 equal Parts, or Degrees, by which the Quantities of all Angles are measured.

PRACTICE.

1. Describe a Circle of the given Magnitude, as *a c e d*, and through its Center *b* draw the two Diameters *c e* and *a d* at right Angles to each other by *Problem* 1. then will the Circumference be divided into four equal Parts, at the Points *c a e d*; and consequently the Circle into four Quarters, each of which is called a Quadrant, as *a b c*, or *a b e*, &c. *Fig.* 25.

2. Make *a* 3, and *e* 6, each equal to the Radius *b e*, or *a b*, then will the Arch *a e* be divided into 3 equal Parts, each = 30 Degrees. And here observe, that as this Division of the Arch *a e* was made by

the Radius *a b*, being ſet from *a* to 3, and from *e* to 6; therefore 'tis plain, that as thereby the Arch is divided into 3 equal Parts, each containing a Third of 90 Degrees, the Radius *a b* muſt be = 60 Degrees of the Arch *a e*. Therefore hereafter, when the Radius of a Circle is mentioned, the Arch of 60 Degrees is to be underſtood by it at the ſame Time.

3. With your Compaſſes divide *e* 3, 3 6, and 6 *a*, each into 3 equal Parts, and each of thoſe Parts in ten equal Parts, (there being yet no Geometrical Method to divide an Arch otherwiſe) then will the Quadrant *a e* be divided into 90 equal Parts; and conſequently, if the other 3 Quadrants *a c b*, *c b d*, and *b e d*, be divided in like Manner, the whole will be divided into 360 equal Parts, as required.

It is by the Number of Degrees contain'd in every Arch of an Angle, that its Quantity is meaſured.

Thus the Quantity of the right Angle *a b c* is 90 Degrees, the acute Angle 6 *b e* 60 Degrees, and the obtuſe Angle *c b* 6 120 Degrees; that is, the Arch or Quadrant *c a* 90 Degrees, and the Arch *a* 6 30 Degrees: which taken together are = 120 Degrees.

Hence 'tis plain, that all acute Angles contain leſs than 90 Degrees; or a right Angle, and all obtuſe Angles more than 90 Degrees, and leſs than 180 Degrees, or two Quadrants: which taken together are = a Semi-circle. Therefore a Semi-circle contains 180 Degrees.

If from the Points 8 7 6 5 4 3 2 1 of the Quadrant there be right Lines drawn to the Center *b*; and if on the Center *b* another Circle be deſcribed, as the Circle *f h g k*, the Quadrant *h g* will be divided by the Lines 8 *b*, 7 *b*, 6 *b*, &c. into the like Number of equal Parts or Degrees, as the Quadrant *a c*; wherefore 'tis plain, that all Circles, whether great or ſmall, have their Circumferences alike divided into 360 equal Parts or Degrees, and each of thoſe Degrees are ſuppoſed to be again ſubdivided into 60 equal Parts called Minutes.

PROBLEM XI.

To make a Line of Chords for the Menſuration of the Quantities of Angles.

PRACTICE.

Fig. 26. 1. Draw a right Line at Pleaſure, as *a b*, and from any Point, as *d*, raiſe the Perpendicular *d n*, and complete the Quadrant *d n b* of any given Magnitude.

2. Divide the Arch *n b* into 90 equal Parts by *Prob.* 10. and then ſetting one Foot of your Compaſſes in *b*, extend the other to the ſe-

veral

veral Divisions of the Arch, and transfer them to the right Line *a b*, as the several prick'd Arches *l* 10, *k* 20, *i* 30, *h* 40, &c. exhibits, which will complete your Line of Chords, as requir'd.

N. B. The larger these Scales are made the better they are for Practice.

PROBLEM XII.

The Angle *b a e* being given to find its Quantity.

PRACTICE.

1. Take 60 Degrees from your Line of Chords, and with that Distance on the angular Point *a* describe the Arch *f e d*. *Fig.* 27.

2. Take the Arch *e f* in your Compasses, and applying that Extent upon your Line of Chords from the Beginning thereof, the extended Point of your Compasses will fall upon the Number of Degrees and Minutes which the Angle contains, *viz.* 60 Degrees, 00 Minutes.

PROBLEM XIII.

To make an Angle of any given Magnitude, suppose 50 Degrees.

PRACTICE.

1. Draw a right Line at Pleasure, as *f d*, then take 60 Degrees from your Line of Chords, and on one End thereof, as at *d*, describe an Arch, as *e c*. *Fig.* 28.

2. Take from your Line of Chords 50 Degrees, the Quantity of the given Angle, and set it on the Arch *e c* from *e* to *b*, then drawing the right Line *d a*, from *a* through *b*, it will complete the Angle required.

PROBLEM XIV.

To describe a Circle whose Circumference shall pass through any three given Points, provided that they are not in a right Line, as the Points *a b c*.

PRACTICE.

1. Draw two right Lines from *a* to *b*, and from *b* to *c*, or from *a* to *c*, it matters not which; then divide those two right Lines contain'd between the three Points, each into two equal Parts by the Perpendiculars *g h* and *f e*, which will intersect each other in *d*; the Centre of the Circle that will pass through the Points given. *Fig.* 29.

2. Set your Compasses in *d*, and extend the Foot to *a*, and then describe the Circle required.

PRO-

PROBLEM XV.

Fig. 30. To defcribe the Equilateral Triangle *a d e*, whofe Sides feverally fhall be = the given Line *f b*,

PRACTICE.

1. Make *d e* = the given Line *f b*; and on the Points *d* and *e* with the Opening or Radius *d e*, defcribe the Arches *b b*, and *c c*, interfecting each other in *a*.

2. Join *a e*, and *a d*, and the Triangle will be compleated as required.

PROBLEM XVI.

To defcribe an Ifofeles Triangle, whofe equal Sides fhall be each equal to a given Line, as alfo its unequal Side to another given Line.

Fig. 31. Let each of the equal Sides be equal to the Line *d d*, and the unequal Side to the Line *e e*. But here Note, That always the Sum of the two equal Sides, muft be greater than the other third Side, or otherwife they cannot form a Triangle.

PRACTICE.

1. Make *c b* = the given Line *e e*, and on the Points *c* and *b*, with the Radius *d d*, defcribe the Arches *g h*, and *f f*, interfecting in *a*.

2. Join *a c*, and *a b*, and the Ifofeles Triangle is completed as required.

PROBLEM XVII.

To defcribe the Scalenum Triangle *a b c*, whofe Sides fhall be equal to the three given Lines *e c*, *f f g g*.

PRACTICE.

Fig. 32. 1. Make *b c* = *e e*, and on *c b* with the Diftance *f f* defcribe the Arch *a a*, and on *b*, with the Diftance *f f*, the Arch *n n*, interfecting *a a* in *d*.

2. Join *d b*, and *d c*, and the Scalene Triangle will be completed as required.

To

PROBLEM XVIII.

To describe the Geometrical Square *a d e f*, whose several Sides shall be equal to the given Line *g h*.

PRACTICE.

1. Make *e f* = *g h*, and on *f* erect the Perpendicular *f d* = to *g h*. Then on the Points *e* and *d*, with the Distance *g h*, describe the Arches *c c* and *b b*. *Fig.* 33

2. Join *a e*, and *a d*, and they complete the Geometrical Square *a d e f*, as required.

PROBLEM XIX.

To describe the Oblong, or Parallelogram *a d f e*, whose Length shall be = the given Line *g h*, and Breadth to the given Line *i k*.

PRACTICE.

1. Make *f c* = *g h*, and on *e* erect the Perpendicular *e d*, = *i k*; then on *d* with the Radius *g h*, describe the Arch *b b*; and on the Point *f*, with the Radius *i k*, describe the Arch *c c*. *Fig* 34.

2. Join *a f*, and *a d*, and they will complete the Oblong, or Parallelogram *a d f e*, as required.

PROBLEM XX.

To describe the Rhombus *a b c d*, whose Sides shall be each = the given Line *e f*.

PRACTICE.

1. Make *a d* = *e f*, and on the Point *d*, with the Radius *a d*, describe the Arch *a b c g*, and make *a b*, and *b c*, each = *a d*. *Fig.* 35.

2. Join *b a*, *b c*, and *c d*, and the Rhombus will be completed, as required.

PROBLEM XXI.

To describe the Rhomboid *a b c d*, whose Angle at *c* shall be = the given Angle *h g i*; its longest Sides each = the given Line *k k*, and its shortest Sides each = the Line *l m*.

PRACTICE.

1. Make *c d* = *k k*, and by *Prob.* 8. make the Angle *c*, = the Angle *h g i*, and make *c a*, = *l m*. *Fig.* 36.

2. On

2. On *a* with the Radius *c d* describe the Arch *e e*, and on *d* with the Radius *c a*, describe the Arch *f f*, intersecting the Arch *e e* in *b*.

3. Join *a b*, and *b d*, and the Rhomboid is completed, as required.

PROBLEM XXII.

To describe the Trapezia *a c f e*, whose Angle at *f*, shall be = the given Angle *i k l*, and several Sides to the four given Lines *k k*, *l l*, *m m*, and *n n*.

PRACTICE.

Fig. 37. 1. Make *f e* = *k k*, and by *Prob.* 8, make the Angle at *f* = to the Angle given *i k l*, and make *f a* equal to *n n*.

2. On the Point *e*, with the Distance of *m m*, describe the Arch *d d*, and on the Point *a*, with the Distance *l l*, describe the Arch *b b*, intersecting the former in *c*.

3. Join *a c*, and *c e*, and the Trapezium will be completed, as required.

PROBLEM XXIII.

To describe any regular Polygon, suppose the Pentagon *a b c d e*.

RULE.

Fig. 38. Divide the Circumference of the Circle, *viz.* 360 Degrees by (5) the Number of Sides contain'd in the Polygon, and the Quotient, is the Number of Degrees contain'd in the Arch of one Side. So 360 being divided by 5, the Quotient is 72. Then taking 72 Degrees from your Line of Chords, set that Distance from *a* to *b*, from *b* to *c*; from *c* to *d*; from *d* to *e*; and from *e* to *a*; and then join *a b*, *b c*, *c d*, *d e* and *e a*, the Polygon required.

LECTURE III.

Of the five Orders of Columns in Architecture, according to the Proportions of the Celebrated ANDREA PALLADIO.

A. WHY do you make Choice of the Proportions of *Andrea Palladio* ?

B. As being the most Grand and Beautiful, and practic'd by most Workmen.

A. What

p 17

Plate II


Tuscan
Dorick
Ionick
Corinthian
Composite
Conteral line
Conteral line
Conteral line
Conteral line
Conteral line

A. What Scale do you uſe, to draw his Columns by?

B. The Diameter of the Column at its Baſe, divided into 60 equal Parts, which I call a Module, and the equal Parts I call Minutes, by which I ſet off the Heights and Projectures of every Member from the centeral Line of the Column.

A. Is it a difficult Thing to draw a Column?

B. No, 'tis very eaſy. I will give you an Example of the Baſe of a Column, by which you will be inform'd of the Nature of deſcribing any other Part, the Rule being the ſame. Plate 2. *Fig.* 1.

Let the Baſe A B C D E F G K be given, to be deſcrib'd in Profile.

PRACTICE.

1. Let L N be the Diameter of the Column given, divided into 60 Minutes as aforeſaid.

2. Draw a right Line at Pleaſure, as *w k*, and in the midſt thereof raiſe the Perpendicular K L, which let repreſent the centeral Line of the Column.

3. Take from your Module 10 Minutes, and ſet it on the centeral Line from K to G; alſo take 7 Minutes and half, and ſet it from G to F; alſo take 1 Minute, and ſet it from F to E; and ſo in like manner, the Diſtance E D 5 half Minutes; the Diſtance D C 1 Minute; the Diſtance C B 5 Minutes; and the Diſtance B A 1 Minute.

4. Thro' all theſe ſeveral Points, draw right Lines parallel to the Baſe *w k*.

5. Make K k, K w, G i, and G v, each = 40 Minutes, and join *v w*, and *i k*, and ſo will the Plinth be completed.

6. Make F r, F g, E q and E f, each = 35 Minutes, and join *q r*, and *f g*, ſo will the lower Fillet be completed; and if from the ſaid Fillet, you deſcribe the Semi-circles *r s t*, and *g h i*, they will complete the lower Torus.

Laſtly, Proceed to ſet off the other Fillets *n d p e*, and *l m a b*, with the upper Torus *m b n d*, and Scotia *p e g f*, the whole Baſe will be completed, as required.

Thus you ſee how eaſy it is to delineate the Baſe of any Order by the Proportions fix'd thereto, and in the ſame manner you may likewiſe complete an entire Order, as exhibited in Plate IV, to which I refer you.

LECTURE VI.

Of the Mechanicks, or the manner of raising heavy Bodies.

1st, *Of MOTION.*

A. WHAT is Motion?

B. The Opposite to Rest. It is two-fold; that is, it is either General or Particular, and those Regular and Irregular.

Motion in General is the Change of a Thing; and when that Change is made in the Substance of the Thing, it is either *Generation* or *Corruption.*

But when that Change is made in the Quantity of the Thing, it is call'd an *Encrease*, or *Diminution.*

And again, When that it is made in Respect to Place, it is call'd Place-Motion, or Local Motion. Wherein observe,

That	Generation, Corruption	*Belongs to*	*Physick.*
	Encrease, Diminution		*Geometry.*
	Local, or Place-Motion		*Mechanicks.*

Which last I shall in a particular manner explain.

A. Sir, I thank you. Pray proceed.

B. Local Motion is the Change of Place, or it is the continual Passage of a Body that moves from one Place to another, as the Passage
Plate 2. *Fig.* 29. of the Body A from the Place B, unto the Place C. For by its being mov'd to C it has chang'd its Place from B to C.

Secondly, If the Body A, as it moves to C, go thro' equal Spaces in equal Times, then its Motion is said to be equal. That is, if B *d* is = *d e*, and the Body A pass from B to *d*, in the same Time as it doth from *d* to *e*, then it will have pass'd thro' equal Spaces in equal times, whereby its Motion is said to be regular or equal.

But had the Body A mov'd from *d* to *e* in less Time than it did from A to *d*, then its Motion had been irregular; because it would have pass'd thro' equal Spaces in unequal Times.

Hence, (as *Galilæus* observes) an irregular Motion is natural to all heavy Bodies, which he justly terms, *A Motion uniformly accelerated*, as a Body dropt from the Top of a Steeple to the Earth, which in equal Times passes through unequal Spaces. That is to say, That dividing the Time it takes up in falling into equal Spaces, as Minutes, Seconds,

conds, &c. The Velocity of the falling Body, at the End of the second Minute, &c. is double of what it was at the End of the first, being reckon'd from the Point or Beginning of its Rest or Fall. And that the Velocity which it acquires in the third Minute, &c. is triple of that which it had the first And the Velocity of the fourth Minute, &c. four times that of the first, and so on in like Proportion of all others. Plate 2. Fig. 2.

Thus, if in the first Minute a Body falls from *a* to *b* in the second Minute, it will have fell to *c*, and have pass'd thro' three times the Space of *a b*, which with the Space *a b* is = 4, which is the Square of 2, the Number of Minutes. *Again*, At the End of the third Minute, it will have fell to *d*, and have pass'd thro' five times the Space *a b*, which with the Space *a b* and *b e* is = 9, which is the Square of 3, the Number of Minutes or Time of falling; and so in like manner of all other Minutes, &c. Hence it follows, that the Spaces thro' which Bodies fall, are, as the Squares of the Times or Minutes, &c. in falling: That is, if in one Minute a Body falls one Foot.

	Min.		*Feet.*	
Then in	2	It will have fell	4	from the Point of Rest.
	3		9	
	4		16	
	5		25	
	6		36	
	7		49	
	8		64	
	9		81	
	10		100	
	&c.		&c.	

Hence it is evident, that the Increase of Motion in every Minute, &c, is according to the Series of the uneven Numbers 1, 3, 5, 7, 9, 11, &c. which are the Differences of the Squares 1, 4, 9, 16, 25, 36, &c.

By the 4th *Prop. 6 Lib. Euclid*, Similar Δs are to one another, as the Squares of their Homologous Sides; one may consider the Spaces pass'd thro' in equal Minutes, as similar Δs, and the Minutes and Velocities as the Homologous Sides of those Δs.

This is easily understood by the Δ A B C, which we suppose to be the Space pass'd thro' by the Body falling; which for Examples Sake we'll suppose has fallen in four Seconds of Time, whose Mea- Fig. 3.

 sure

ſure ſhall be repreſented by the Side A B, equally divided at D E F B, into four equal Parts; and the Baſe B C ſhall likewiſe repreſent the Velocity, which the Body has acquir'd in falling.

Now, as each of the equal Parts A D, D E, E F, and F B, repreſent one Second of Time; ſo likewiſe if B C be divided into four equal Parts, as B G, G H, H I, and I C, each of thoſe Parts will repreſent one Degree of Velocity, becauſe 'tis ſuppos'd that the Velocities and Seconds encreaſe continually in the ſame Proportion.

Again, If from the Points I H G, you draw right Lines parallel to A C, and to A B alſo, interſecting A B, in the Points D E F, and A C in the Points M L K, the Triangle A B C will be divided into ſixteen little Triangles = one another, and each ſimilar to A B C.

Now, Since that A D repreſents the firſt Second of Time, and D M, the firſt Degree of Velocity; therefore the Triangle A D M, will repreſent the Space which the Body has paſs'd thro' in the firſt Second, with one Degree of Velocity. So likewiſe, the Line A E repreſenting the ſecond Second of the Fall of the ſaid Body, the Line E L will repreſent the Velocity which the Body has acquir'd in falling the ſecond Second of Time, and the Triangle A E L, will repreſent the Space that the Body has paſs'd thro' with two Degrees of Velocity, which Triangular Space A E L is = four times A D M. Becauſe the ▽ *a* is = the ▽ *m*, and the ▽ *m* is = the ▽ *n*, and the ▽ *n* is = the ▽ *o*. Therefore the ▽ A E L is = 4 times the A D M, ▽ and ſo in like manner of all other equal Spaces of Time.

Hence 'tis evident, That the Velocity with which heavy Bodies deſcend, is according to the Squares of their Times.

Sir, I perfectly underſtand you; That whatever Space a Body paſſes thro' by falling in one Second, or one Minute, or one Hour, ſo many times that firſt Space, the Body falls as are = the Square of its Times. That is, If in one Minute a Body falls one Foot, then in ten Minutes Time it will have fell one hundred Foot, and in twelve Minutes 144 Foot, becauſe 100 is the Square of 10 multiplied by 10, and 144 is the Square of 12 multiplied by 12.

B. You are right. Now I will proceed to inform you, in the ſecond kind of Motion, namely Vibration.

Plate 2. Fig. 4.

Secondly, Of V I B R A T I O N.

Vibration is the circular Motion of a Body, as B or C, ſwinging on a Line, *&c.* faſtned at A as a Center, which Point A is call'd the Center of Motion, and by ſome, the Center of reciprocal Motion; the Point

D is

D is call'd the Point of Reſt, and the Line A B, taken with the Body B, is call'd a Pendulum.

A. Pray why is A call'd the Center of reciprocal Motion?

B. Becauſe when the Pendulum A D, is mov'd from the Point of Reſt D to C, it moves about that Point A, to return to D, firſt on one Side, then on the other, until by its own Gravity, it ceaſes its Motion, and remains in D the Point of Reſt: Wherefore 'tis call'd the Center of reciprocal Motion.

Vibration, is either Simple or Compound; that is Simple when the Pendulum has mov'd from B C, and Compound, when it has return'd back again from C to B, *&c.*

Pendulums of equal Lengths and Weights, whether great or ſmall, perform their Vibrations very near in the ſame Time. But Pendulums of different Lengths will vibrate unequally, becauſe a longer Pendulum muſt remove more Air in its Swing or Vibration than a ſhorter.

It has been found by ſeveral Experiments, that the Length of two unequal Pendulums are reciprocally proportionable to the Squares of the Numbers of their Vibrations in an equal Time; as the Squares of their Vibrations. That is, the Length of the firſt Pendulum: is to that of the Second:: As the Square of the Numbers of the Vibrations of the Second: in a given Time: Is to the Square of the Numbers of the Vibrations of the firſt in the ſame Time. *Fig.* 5.

Mr. *Henry Phillips*, in his Advancement of the Art of Navigation, affirms, That if a Pendulum be made = 38 Inches and half from A, the Center of Motion, to C, the Center of Gravity of a Bullet, *&c.* every Vibration of ſuch a Pendulum will be = one Second, or 60th Part of a Minute of Time: That is, every time that the Body C or D paſſes by the Point of Reſt B, either from B to C, and back again to B, or from B to F, and back again to B, will be = one Second of Time, and conſequently its Motion from C to B, or from B to F, *&c.* muſt be = half a Second of Time.

And here Note, That it matters not, what Swing or Diſtance from the Point of Reſt, you firſt give it; for a Body will vibrate in the ſame Time from C to F, as from D to E.

Therefore, if ſeveral Pendulums of equal Lengths and Weights, were ſet going together at the ſame Time, with different Forces given them at firſt, they would be all in perpendicular Poſition, as A B, at the ſame Time. For tho' the Body C being rais'd higher from B, than the Body D, will vibrate with greater Velocity than the Body D, which

which is rais'd but half the Height of C. Yet if both Sides are ſet going at the ſame Time, they will paſs by the Point of Reſt at the ſame Time, for their Velocity being proportionable to the Spaces through which they paſs. This is plain, for as the Body D vibrates but to E, which is but half the Arch C F, through which the Body C doth vibrate at the ſame Time; therefore the Body D requires but half the Velocity of the Body C, *&c.*

A. 'Tis very plain *Sir*: By theſe Pendulums I can eaſily meaſure Time at my Pleaſure, for ſince that one Vibration is = 1 Second of Time, therefore 30 is = half a Minute, and 60 to one whole Minute; and beſides, their other Properties are vaſtly pleaſant, and I hope uſeful alſo, as I go on in Mechanicks. Pray *Sir*, proceed to your next Definition.

B. The next in Order is, *Heavyneſs of Matter*, which is alſo call'd *Weight* and *Gravity.*

It is the natural Inclination which is in heavy Bodies, to move downwards, when they are not ſuſtain'd or held up, and fall towards the Center of the Earth.

And it is for this Reaſon, that the Center of the Earth is by ſome call'd, *Centrum Gravium*, or the Center of Heavy Bodies. But whether the Center of Gravity and Center of the Earth, is one and the ſame Point, I believe that none can prove.

The Center of Gravity of a heavy Body, is a Point, by which a Body being ſuſpended, all its Parts, which are about that Point, will ballance one another, and oppoſitely hinder one another from falling, whereby the Body will remain in any given Poſition. Wherefore it is plain, that a Liquid Body cannot of its ſelf have any Center of Gravity, becauſe its Parts are not fix'd to one another; but are in a continual Motion, as Water, Wine, Beer, *&c.*

A. Pray why do you doubt that the Center of the Earth's Gravity, is not the ſame as the Center of its Magnitude.

B. The Centers of Gravity and Magnitude cannot be in the ſame Point, but in a Body which is *Homogeneous.* That is, a Body is call'd *Homogeneous*, whoſe Matter is uniform, and every where of the ſame Weight about its Center of Magnitude, which is a Point in that Body, as far diſtant as can be, and equal from all its Extremities, which the Matter that compoſe our Earth is known to be otherwiſe, ſome being of Earths, Metals, Minerals, Water, *&c.* whoſe ſpecifick Gravities have very great Differences, and therefore the Earth, or any other Body, whoſe Parts or Matter are of different Weights in different Parts, are call'd *Heterogeneous* Bodies.

A. Pray

A. Pray what do you mean by ſpecifick Gravity?

B. The ſpecifick Gravity of a Body, is that which proceeds from the natural Deſinity of the Parts of its Matter, which makes one Body weigh more than another of the ſame Dimenſions or Magnitude. As for Example. The ſpecifick Gravity of Water is greater than that of Oil; that of Gold, is greater than that of Lead, *&c.* as exhibited in the following Table of Sir *Jonas Moor.*

A Table exhibiting the Weight (Averdupois) of a Cubical Foot, and Inch of the following Metals, &c. *the former in Pounds and Decimal Parts of a Pound, and the latter in Ounces and Decimal Parts of an Ounce.*

			Pounds.	D. Pts.		Ounces.	D. Pts.
A Cubical Foot of	Fine Gold	Weights	1352	4	And one cubical Inch	11	365
	Standard Gold		1180	4		10	930
	Quick-Silver		874	9		8	101
	Lead		707	7		6	553
	Fine Silver		693	1		6	418
	Standard, *Do.*		658	3		6	096
	Copper		562	4		5	208
	Braſs		521	8		4	832
	Caſt Braſs		500	0		4	630
	Steel		490	7		4	544
	Iron		477	5		4	422
	Tin		457	4		4	236
	Marble		169	3		1	568
	Glaſs		161	2		1	493
	Allablaſter		117	0		0	
	Ivory		113	9		1	55
	Sea-water		64	1		0	594
	Common Water		62	3		0	578
	Dry Oak		57	8		0	536
	Olive Oyl		57	0		0	528

A cylinderical Foot of	Sea-water	Weights	50	481	
	Com.-water		49	106	
A cylinderical Inch of	Sea-water	Weights	467		Ounces.
	Com.-water		454		

The

The ſpecifick Gravity of heavy Bodies is either *abſolute* or relative.

1. The abſolute Weight of a heavy Body is the Force which it has to deſcend freely in a fluid Medium, as in Air or Water, when it touches nothing elſe but the Parts of that Medium: Thus the Weight of a Stone which is in the Air is called abſolute, from its own Force which it has to deſcend freely, when it touches nothing but the aireal Particles thro' which it falls.

2. The relative Weight of a heavy Body (called by the *Greeks* ῥοπὴ, and by the *Latins Momentum*) is the Force which ſuch a Body has to deſcend when it touches ſomething elſe, more than the Parts of

Fig. 6. the *Medium*, as when it bears on an inclined Plain, as A on B, or on

Fig. 7. the End of a Leaver, as the Body E on the Leaver F G, where it often happens that the Body in Queſtion becomes a Counterpoiſe to a greater and heavier Body, as the Body C, as it is nearer or farther from the Centre of Motion D, on which the Leaver moves.

This Counterpoiſe of Bodies is called *Equilibrum*.

Fig. 6. Now 'tis plain that the Body H, which is ſuppoſed = the Body A, in falling to K, muſt fall with greater Force than the Body A, becauſe that the Body H has no other Reſiſtance than that of the Air: but the Body A has the Reſiſtance of the inclined Plain K I and Air alſo. Therefore 'tis evident, that the abſolute Weight of any Body is greater than the relative Weight.

Fig. 7. *A.* Pray Sir, pardon me for interrupting you, and be pleaſed to tell me what you mean by the Centre of Motion (D of the Leaver F G).

B. The Centre of Motion in a Leaver is that Point whereon it reſts and moves, as D; in a Ballance it is that Point which it hangs by, as the Point B of the Ballance A C. and in a heavy Body it is

Fig. 9. that Point by which a Body is held, and about which it may be mov'd,

Fig. 8. as the Points or Centers of the Circle, Square and Equilateral △. D E F

This is alſo by ſome called the fix'd Point, and by the *Greeks* ὑπομόχλιον, or propping Point; and by the *Latins Anſa* or *Fulcrum*, which laſt is moſtly uſed by Engineers and Workmen.

Now, as I have thus given you ſome neceſſary Definitions of Motion, I ſhall, in the next Place, proceed to inform you in like Manner of *Power*, by which all Bodies are moved and raiſed by the following Engines, and the Application thereof in Practice.

1. A POWER *is whatever can move a heavy Body*, and is therefore alſo called the *moving Force.* Thus Weight is a Power, in Reference to a heavy Body which it may move.

Power

Power is two-fold, that is, either animate, as the Power of a Man, Horſe, &c. in pulling, drawing, &c. or inanimate, as the ſpecifick Gravity of a Body of Gold, Iron, Stone, Water, &c. as one Pound, ten Pounds, &c. Weight.

The Quantity of Power is eſtimated by the Quantity of the Weight of the Body which it ſuſtains, that is, when a Power ſuſtains twice or thrice its own Weight; then we ſay that, that Power is double or treble that Weight which it doth ſuſtain.

2. The Manner of applying a Power to a Leaver may be immediately on the Leaver as a Weight E laid on the End of the Leaver G F, *Fig.* 7. or at ſome Diſtance from it, as the Weight D hung on the Point C, by *Fig.* 10. Means of the Chord D C, and that right Line in which a Power or heavy Body endeavours to move in, is called the Line of Direction of that Body; ſo C I is the Line of Direction of the Body D, and A Z, of the Weight I.

The real Application of a Power to a Leaver is that Angle which is conſtituted by the Leaver and Line of Direction at their Point of meeting; thus the Angle A B E conſtituted by the Line of Direction E B and Leaver A B is the Application of the Power E; ſo likewiſe *Fig.* 11. are the Angles A B F and A B G Applications of the Powers F and G.

A. I underſtand you. But pray Sir, ſuppoſe that the Powers at E F and G are = on one another, that the Line of Direction of the Power F makes an Angle with the Leaver of 90 Degrees; and the other Powers E and G at equal Diſtances from F. Is their Effects or Powers of raiſing the Weight D in them three ſeveral Stations = one another?

B. No, the Power F, which is apply'd to the Leaver B at right Angles, hath the greateſt Effect, not only of the other two Powers E and G, but of all others which are not perpendicular to the Leaver A B.

A. How do you prove it?

B. As following. 1. The Diſtance of a Weight, or of a Power from the *Fulcrum*, is the neareſt Diſtance contain'd between the *Fulcrum* and Line of Direction; that is, it is a right Line or Perpendicular let fallen from the *Fulcrum* upon the Line of Direction, as C F on the Line of Direction B E. 2. If on C with the Radius C F, you deſcribe the Arch F B, 'tis evident that C K is leſs than C B, and the Point K is nearer to the *Fulcrum* C than the Point B: and ſince that the farther the Power is apply'd from the *Fulcrum* the greater Force it will have; therefore 'tis evident that the Power F

which acts upon the Part of the Leaver B, muſt have greater Force than the Power E, whoſe Diſtance from the Fulcrum is = C K, which is leſs than C B.

A. Sir, I conceive it, and from thence it appears, that the leſſer the Angle of Application is, the greater the Power muſt be increas'd to become = the Power F, which is applied at right Angles. And I ſuppoſe that the greater the Angle of Application is made, as the Angle C B G, the leſſer the Force is requir'd to be = the Power F.

B. Your firſt Obſervation on the Powers applied with Angles acute is juſt, but your Suppoſition of the obtuſe Angles requiring a leſſer Force to equalize F is falſe; which I'll thus prove.

1. It has been already ſaid, that the Diſtance, of a Power from the Fulcrum, is a right Line or Perpendicular, let fall from the Fulcrum upon the Line of Direction.

2. Since the Leaver C B, is the Perpendicular its ſelf, to the Line of Direction B F of the Power E, whoſe Angle C B F is a right Angle, 'tis evident, that if the Power F be remov'd to G, then the Angle C B G will be an obtuſe Angle. And ſince, that when any right lin'd Triangle hath one of its Angles obtuſe, the Sum of the other two muſt be leſs than a right Angle, becauſe the Sum of all the three Angles taken together, are always = 2 right Angles or 180 Degrees.

3. Now ſince that the Angle C B G is an obtuſe Angle, it is therefore impoſſible that a Line can be drawn from the Fulcrum C to the Line of Direction B G, and to be perpendicular thereto alſo.

But to ſupply this Defect, you muſt continue on the Line of Direction G B thro' the Point of Application B upwards towards H; and then if you let fall a Perpendicular Line from the Fulcrum C to the continu'd Line of Drection B H, it will cut the Line B H in H, then C K.

4. If the Diſtance of the Power G from the Power F be = the Diſtance of the Power E from the Power F, then will the Perpendicular C H be = to the Perpendicular C E, and therefore the Power applied at G whoſe obtuſe Angle C B G exceeds the right Angle C B F, as much as the acute Angle C B E is leſs than the right Angle C B F; is equal in Force to the Power E, and both leſs in Force than the Power E, Q E D. Hence 'tis evident, that if the Power G was to thruſt, or preſs at H on B, its Force would be the very ſame, as when pulling at G, and that when Workmen apply their Strength to raiſe up heavy Weights, they ſhould always endeavour to apply the ſame as near to a right Angle with the Leaver as they poſſibly can.

A. 'Tis

A. 'Tis very true *Sir.* But pray before you proceed further, resolve me another Question, which is this; Whether or no, if a Power as P being hung close to the Leaver A D has not a greater Force than when hung on the same Point D at the End of a long Cord or Line, as the Weight E. *Fig.* 12.

B. No *Sir*, if the Bodies P and E are equal, the Body E will have the same Force as the Body P, and if the Gravity or Weight of the Cord be consider'd and added thereto, it will have a greater Force than the Body P, for was you to sustain the Weight E by the End of the Cord at D, you must at the same Time sustain both their Weights.

A. I ask Pardon, but such is the Opinion of many as it was mine, for want of knowing better.

B. Sir, It is your Place to ask what you desire to know, and mine to instruct you in the Truth thereof.

A. Pray *Sir*, is there any other Thing to be learn'd before I proceed to the Practice of the Mechanick Powers?

B. Yes, I must first give you a Word or two more concerning the natural Descent of heavy Bodies, and of their Line of Direction, in which they endeavour to descend.

A. Pray proceed?

B. A heavy Body naturally descends to the lowest Place that it can go, provided that its Descent is not oppos'd by any other Heavy Body.

And as all the Parts of *Homogeneous* Bodies have an equal Pressure about their Centers of Gravity, therefore the chief Endeavours of Bodies to descend, is made by the Descent of their Centers of Gravity. For if the Center of Gravity of a Body, do not descend, but remain fix'd, the whole Body will remain fix'd also, because it is to the Center of Gravity that all the Parts of the Body has a close Adherence.

Hence its plain, that the inclin'd Body C D B A, *Fig.* 13. cannot fall towards F which it inclines to, because its Center of Gravity E must be oblig'd to ascend, and pass thro' the Arch E P, which it cannot do, the Part or Quantity G C A B standing over the Base, wherein the Center of Gravity is, being greater than the inclining Part G D A. Therefore 'tis evident, that no such Body can descend, when the Line of Direction, or Center of Gravity doth not exceed the Extreams of the Base B A. *Fig.* 13.

And on the contrary, when the Center of Gravity of an inclin'd Body, as E, *Fig.* 14. exceeds the Limits of the utmost Perpendicular as G A,

Fig. 14. G A, whereby the Part ſtanding over the Caſe G B A, is leſſer than the inclining Part G A C D: Then ſuch Bodies will fall, for the Center of Gravity E, having A for its Center, will freely deſcend in the Arch E F.

Now 'tis very plain, that to have a Body remain ſtedfaſt upon its Baſe, and is not inclin'd, the Line of Direction muſt of Neceſſity fall in ſome Part of the Baſe of the ſaid Body, or otherwiſe it will naturally fall.

Whence it follows, that the leſſer the Baſe of any upright Body is, the eaſier it will move out of its Poſition, becauſe the leaſt Change is capable of removing the Line of Direction out of its Baſe.

This is the Cauſe why a Ball, or Sphere, whoſe Baſe is a Point only rowls eaſily on a plain Superfices by a gentle Force.

This Law of Mechanicks is obſerv'd by every Animal in their riſing and ſtanding, to prevent their falling; as for Example in humane Bodies, when we are to riſe from a Seat, we naturally bend our Bodies forwards, ſo as to cauſe the Line of Direction of our Bodies to paſs through our Feet, upon which we bear our ſelves when we begin to aſcend.

Now from the preceeding it follows, That the wider the Baſe of any Body is, the eaſier it will ſupport itſelf, becauſe then the Line of Direction cannot go out of the Baſe without greater Force.

This being well underſtood, will be of very great Service to Painters, Carvers, and Statuaries, in giving their Figures ſuch Poſtures as are agreeable to Nature; as alſo to the Maſons, Bricklayers, &c. in proportioning the Thickneſs of Walls, according to their ſeveral Heights required.

I ſhall now conclude this Lecture with obſerving to you, that all the Powers or Bodies produced in the following Lectures, are ſuch that doth equipoiſe each other, or are equal in Power to each other, according to their ſeveral Ratio's. And therefore obſerve,

That when a Power can ſuſtain a Weight by the means of any Ballance, Leaver, &c. a Power greater, as little as can be imagin'd, will over-poiſe, or cauſe the ſaid Weight to riſe.

Likewiſe note, that the Weight of the Leavers, Pullies, &c. and their Friction, is not conſider'd. A Leaver being conſider'd as a right Line, and a Pully as moving on a real Point.

LECT.

Plate II
p. 29
Fig I
Fig II
First Min
Second Min
Third Min
Fourth Min
Fifth Min
Sixth Min
Fig III
Fig IIII
Fig V
Fig VI
Fig VII
Fig VIII
Fig IX
Fig X
Fig XI
Fig XII
Fig XIII
Fig XIIII
Fig XV
Brachia Major
Brachia Minor
Fig XVII
Fig XVIII
Fig XIX
Fig XX
Fig XXI
Fig XXII
Fig XXIII
Fig XXIIII
Fig XXV
Fig XXVI
Fig XXVII
Fig XXVIII
Fig XXIX

LECTURE V.

Mechanick Propositions.

Of the BALLANCE.

FIRST note, that A C *taken together, are call'd the Beam of the Ballance.*

The Point B, *the fix'd Point on which it moves and equally divides* A B, *and* B C, *which are call'd two Brachia's.*

THEOREM.

If two Weights ty'd to the Ends of an horizontal Ballance, are to one another reciprocally as their Distance from the fix'd Point, they shall hang in *Equilibro.* Plate 2. Fig. 9.

Of equal Brachia's.

DEMONSTRTION I.

Let A B = B C, and the Weight D = E. Then I say,

As D 2 : E 2 : : A B : B C.

Or, As D 2 : B C : : E 2 : A B, Q E D

DEMONSTRATION II.

Of unequal Brachia's.

1. Let B be the fix'd Point.
2. Let A C be = 24 Feet.
3. Let A B = 8, and B C = 16 Feet.
4. Let the Weight D = 12 Pound and E = 6 Pound.

Then I say,

As the lesser Brachia A B 8, is to the lesser Weight E 6,

So is the greater Brachia B C 16, to the greater Weight D 12, which is required to equipoise E. Fig. 15.

Or, As the lesser Brachia A B 8.

Is to the greater Brachia B C 16.

So is the Power apply'd at C, *viz.* E 6. to the Weight that it will equipoise at A, *viz.* D 12. Q E D.

PROPOSITION II.

Knowing the Weight of two heavy Bodies, applied to the Ends of a Ballance of a known Length, to find upon that Ballance the common fix'd Point or Center of Motion, whereon the two given Bodies will hang in *Equilibro.*

DEMON-

DEMONSTRATION.

Let the Ballance A B be = F 24 Foot.

The Weight D = 12, and the Weight E = 6.

Then I say

As the Sum of both Weights 18, is to the lesser Weight D 6.

Fig. 15. So is the whole Ballance A C 24 to the lesser Brachia A B 8.

Or as the Sum of D + E is to A C : :

So is the greater Weight D 12, to the greater Brachia B C 16. Therefore 'tis evident, that the Point B is the fix'd Point or Center of Motion requir'd — Q E D.

N. B. It is here suppos'd, that the Ballance A C is without Weight in its self, as a Line *&c.* —— as before noted.

PROPOSITION III.

The Length and Weight of a Ballance given, which has at one of its Ends a Body of known Weight to find the fix'd Point about which the Weight of the Ballance and the Weight of the Body shall remain in *Equilibro.*

1. Let the Ballance weigh 16 Pound, and its Length 12 Feet.
2. The Body E 8 Pounds.

Then I say

Fig. 16. As 24 the Sum of the Weights of the Ballance 16 Pounds, and the Body 8 Pounds taken together ;

Is to the whole Length of the Ballance 12 Feet.

So is 8 the Weight of the Body E, to the lesser Brachia A B 4.

Or thus,

As 24 the Sum of the Weights of A D + E ;

Is to 12 the Length of the Ballance : :

So is 16 the Weight of the Ballance ; to B D, 8 the greater Brachia. Therefore 'tis evident, that the Point B is the Point required.

PROPOSITION IV.

Two Bodies being given, the heaviest of which hangs at one of the Ends of a Ballance of known Length and Weight, and given fix'd *Fig.* 17. Point (as Steel-yards) to hang that of least Weight in such manner, that being assisted by the Weight of the Ballance, it may keep the heaviest Body in *Equilibro.*

Let the Ballance A B weigh two Ounces, and 14 Inch long.

2. Let C be the fix'd Point, one Inch from the End A, and let the Part D of Body D O weigh 15 Ounces.

3. Let

3. Let E be a Body of one Ounce, and moveable at Pleasure, to find the Point F, where the Body E with the assistance of the Gravity of the Brachia C B, of the Ballance A B, shall keep the Weight D O in *Equilibro*, about the Center of Motion C.

4. Divide the Ballance A B into 2 = Parts at G, which (if made of equal Matter) will be its Center of Gravity.

5. Suppose the Body H in Weight = the Ballance A B, *viz* = 2 Ounces, to hang from the Point G.

Then as the Distance of A C.

Is to the Distance C G.

So is the Weight H = the Weight of the Ballance ; to a 4th proportionable = one part of the Weight D O, *viz.* = 12 Ounces, wherefore the other part thereof remaining is = 3 Ounces.

Now again,

As 1 Ounce the Weight of the Body E is to the Part D 12 Ounces, the last N°. found, so the Distance of A C to 3, the Distance of F from C, whereon the Body E being hung, will keep the Body D O in *Equilibro.*

This being evident, needs no further Demonstration.

LECTURE VI.

Of the LEAVER.

THE Leaver is no other than the Ballance, saving the Manner of its application in Practice. That is, as the Ballance is suspended or hung on the fix'd Point or Center of Motion, as A C on B, the Leaver rests upon a Point, as D F on E, *Fig.* 18, which is also call'd either *the fix'd Point*, *Center of Motion*, *Fulcrum*, or *Fulcimen.*

A. How many kind of Leavers are us'd in Practice?

B. Four, and are each distinguish'd by their kinds, as a Leaver of the first kind, of the second kind, of the third kind, and of the fourth kind.

A. What is a Leaver of the first kind?

B. A Leaver of the first kind, is that whose Fulcrum is between the Power applied and the Weight that is to be rais'd, as A C, where the Power is applied at C, the Weight at A, and the Fulcrum between them, as at B. *Fig.* 19. *Fig.* 19.

A. How shall I know what Weight can be rais'd by this Leaver, with a given or known Power or Strength applied at C?

B. By this Canon or Analogy. As

As the lesser Brachia A B, being always contain'd between the middle of the Leaver and Weight to be rais'd.

Is to the greater Brachia, B C.

Fig. 19. So is the Power applied at C to the Weight that it will raise at A.

Suppose the Leaver A C to be 12 Foot long, and the Power applied = 10 Pound Averdupoise, and let the Fulcrum B be at 9 Feet distance from C. Then I say

As 3 the lesser Brachia,

Is to 9 the greater Brachia,

So is 10 the Power applied at C to 30, the Weight that C will raise at A.

OPERATION.

$$
\begin{array}{l}
3 \quad 9 \quad 10 \quad\quad 30 \\
\qquad\quad\ \ 9 \\
\qquad\ \overline{} \\
3)\ \ 90\ \ (30
\end{array}
$$

And here observe, that the nearer the Fulcrum is plac'd to the Weight, the greater Weight can be rais'd.

For Example,

Suppose that the Fulcrum be plac'd at 10 Feet distance from C *Fig.* 19. at E, then I say.

As A E 2 the lesser Brachia,

Is to E C 10 the greater Brachia.

So is 10 the Power applied at C to 50, the Weight that C will raise at A.

OPERATION.

$$
\begin{array}{l}
2 \quad 10 \quad 10 \quad 50 \\
\qquad\quad\ \overline{} \\
\qquad\quad\ 10 \\
2)\quad 100\ \ (50
\end{array}
$$

Now 'tis plain, that by moving the Fulcrum one Foot nearer towards the Weight, the Power is increas'd from 30 to 50, and therefore to equipoise the Weight A on the Fulcrum E, then is but 6 Pounds required as a Power at C, For,

As E C 10 Feet the greater Brachia

Is to A E 2 Feet,

So is 30 the Weight A; to 6, the Power requir'd at C to equipoise A.

OPERA-

OPERATION.

$$\begin{array}{l} 10 \quad 2 \quad 30 \quad 6 \\ \qquad\quad\;\; 2 \\ \hline 10)\ 60\ (6 \end{array}$$

Hence 'tis evident, that the nearer the Fulcrum is to the Body or Weight, the leſſer the Power is required to equipoiſe the ſame, and conſequently the leſſer to raiſe the ſame; or otherwiſe, the farther the Power is diſtant from the Fulcrum, the more Force it will proportionably have.

But here obſerve, that when by moving the Fulcrum near to the Weight whereby the Power is increas'd, that at the ſame time the Space or utmoſt Height of raiſing the Weight is diminiſh'd accordingly. For Example.

Let *i l* be a Leaver 12 Foot long, with its Fulcrum *k* at 9 Feet from *l*. Then if the Point *l* be depreſs'd to *m*, it will raiſe the Body *i* unto *a*, on the horizontal Line *c d*. But if the Weight or Body *i* be mov'd nearer to *k*, as at *h*, whereby a leſſer Power will raiſe it, then when the End of the Leaver *l* is depreſs'd, as before to *m*, the Body *h* will be rais'd no higher than *b* on the horizontal Line *e f*. And again, had the Body *i* been plac'd at *n*, it could not be rais'd higher than *m* on the Line *o p*, and ſo in like manner of all others. Q. E D. *Fig.* 20.

Hence 'tis plain, that the higher the Body is rais'd, the greater Diſtance it muſt be from the Fulcrum, and conſequently the greater Strength or Power is required to raiſe the ſame.

Whence 'tis evident, that as the Diſtance of the Weight from the Fulcrum may be greater (as B C *Fig.* 21.) than the Diſtance of the Power A B, or leſſer (as A B) than the Power B C, or equal to one another, as A B and B C, in the Ballance, *Fig.* 9. ſo proportionably muſt the Powers be applied.

A. Suppoſe that A E is a Leaver 12 Feet long, that the Fulcrum be fix'd at C 3 Feet from E, the Place where the Power is to be applied, and that the Body F hanging at E weigh 30 Pounds, Pray what Power at A will equipoiſe E, and by what Analogy do you find it? *Fig.* 28.

B. The Analogy is the following.

As C E 3 the leſſer Brachia
Is to C A 9 the greater Brachia,

So is 40 the Weight E to 90, the Power required at E to equipoise the Body F at A.

OPERATION.

$$3 : 9 :: 40 : 120$$

$$\begin{array}{r} 9 \\ \hline 3)\ 360\ (120 \end{array}$$

Hence 'tis plain, that this Analogy is the same as the first Analogy of the Leaver of the first kind, for if you suppose that the Body E be a Power given, then the Power required to equipoise the same, is no more than to find the Weight or Power that the given Weight will equipoise.

So much with relation to the Leaver of the first kind; now I shall proceed to a Leaver of the second kind.

A Leaver of the second kind, is that wherein the fix'd Point or Fig. 22. Fulcrum is at one End (as at A,) the Power applied at the other End, (as at C) and the Weight suspended between them, as at E B F, &c.

A. By what Canon or Analogy do you find the Weight that any given Power will raise; or what Power is requir'd to raise a given Weight?

The following, That is to say,

As the Distance of the Weight from the Fulcrum,

Is to the Distance of the Power from the Fulcrum,

So is the Power to the Weight that will equipoise it.

And here Note, That when the Equipoise of any Weight is found, a very small Addition thereto is the Power that will raise the same.

Let the Power at C be = 10 Pounds Averdupois, and the Leaver A C be = 12 Feet long, and let the Body D be hung in the middle thereof at B 6 Feet distant from the Power C, as well as from the Fulcrum A. Then I say,

As B C 6 Feet, the Distance of the Weight from the Power,

Is to A B 12 Feet, the Distance of the Power from the Fulcrum,

So is 10 Pounds the Power at C to 20 Pounds its *Equilibrum.*

Again,

Let the Body D be mov'd to E at 3 Feet Distance from the Fulcrum A. Then I say,

As A E 3, the Distance of the Weight from the Fulcrum,

Is to A C 12 the Distance of the Power from the Fulcrum,

So is 10, the Power applied at C, to 40 its *Equilibrum.*

OPERA-

OPERATION.

As 3 . 12 10 40
12
———
(3 120 (40

Again,

Let the Body or Weight D be mov'd to F, at 9 Feet Distance from the Fulcrum A. Then I say,

As A F 9 Feet, the Distance of the Weight from the Fulcrum, Fig. 22.
Is to A C the 12 Feet, the Distance of the Power from the Fulcrum,
So is 10 the Power applied at C, to 13 and half its *Equilibrum.*

OPERATION.

9 : 12 :: 10 : 13 + 3 qrs.
12
———
9) 120 (13 3 qrs.

Hence its also evident, as in the Leaver of the first kind, that the nearer the Weight is to the Fulcrum, the greater the Power is increas'd. For in this last Example where the Weight was applied at F, 9 Feet Distance from the Fulcrum A, the Power C 10, could equipoise but 13 Pounds 3 qrs.; but where the Weight was applied nearer to the Fulcrum, as at B, 6 Feet from the Fulcrum A; then its Equipoise was = 20 Pounds. And again, when the Weight was applied still nearer unto the Fulcrum, as at E, then the Equipoise of C was = 40 Pounds. Q E D.

And as I have already prov'd in the Leaver of the first kind, that what is gain'd in Power is lost in Space or Time, so also 'tis the same in this kind of Leaver.

For Example,

Suppose the Power at K is to be rais'd from K to E, = 6 Feet above I, and at the same time was to raise the Weight G plac'd in the midst thereof. Then I say, That tho' the Weight equipoised at C, is double to the Power E, yet C is rais'd but half the Height of E above I. That is, Fig. 23.

As the Equipoise G rais'd to C
Is double the Weight of the Power K rais'd to E,
So is the Space or Arch E K, through which the Power K pass'd in going to E, double or = twice the Space or Arch G C, through which the Body or Weight G pass'd in going to C. and so in like Proportion of all others according to the Distance from the Fulcrum.

Now

Now to find a Power equal to a given Weight, having the Fulcrum assign'd, and the Length of the Leaver given.

This is the Analogy.

As the Distance of the Power from the Fulcrum

Is to the Distance of the Weight from the Fulcrum, so is the given Weight to the Power required to equipose the same.

Let the given Weight D be = 50 Pound, placed at E 3 Feet Distance from the Fulcrum A, and let the Power be applied at C, 12 Feet distant from the Fulcrum A. Then I say

As 12 the Distance of the Power from the Fulcrum

Is to A E 3, the Distance of the Weight from the Fulcrum, so is 50 the given Weight of the Body D to 12 and half, the Power required at C to equipoise D. *Fig.* 24.

OPERATION.

As 12 3 50

$$\begin{array}{r} 3 \\ \hline 12)\ 150\ (12\ \tfrac{1}{2} \\ 12 \\ \hline 30 \\ 24 \\ \hline 6 \\ \hline 6\ 12\ \tfrac{1}{2} \end{array}$$

A. But pray resolve me another Question. That is, when with a Leaver of the second kind I raise a Body of 50 Pounds Weight, with a Power = to 25 Pound, pray what sustains the other 25 Pounds?

B. The Fulcrum on which it rests.

A. Pray how do you prove that?

B. I'll shew you.

Suppose that at A and B were two Powers, sustaining the Weight B at D, 3 Foot from A. Now I say, that as the Weight D is nearer to the Power at A than to the Power at C, that therefore the Power at A sustains the greatest part of the Weight. *Fig.* 24.

DEMONSTRATION.

1. Suppose A to be the only Power and C the Fulcrum, and let the Leaver A C be = 12 Feet

Then I say as before,

As A C 12 the Distance of the Power A from the Fulcrum C

Is to B C 9, the Distance of the Weight D from C,

So is D 50 the given Weight to 37 and half, the Power required at A to equipoise B.

OPERATION.

12 9 50 37 $\frac{1}{2}$

9

12) 450 (37 $\frac{1}{2}$

36

90

84

6 = $\frac{1}{2}$

Hence 'tis evident, that the Power A sustains 37 and half Pounds of the Weight B, which is = 50 Pounds.

Again,

Now suppose A to be the Fulcrum, and C the Power with the Weight as before.

Then I say,

As C A 12, the Distance of the Power C from the Fulcrum A,

Is to A B 3, the Distance of the Weight from the Fulcrum,

So is D 50, the given Weight; to 12 half the Power required to equipoise D.

OPERATION.

12 3 50 : 12 $\frac{1}{2}$

3

12) 150 (12 $\frac{1}{2}$

12

30

24

6 = $\frac{1}{2}$

Hence 'tis evident, that the Power C sustains but 12 half Pounds.

Now if to 37 $\frac{1}{2}$ be added 12 $\frac{1}{2}$, the Sum is = 50, the given Weight sustain'd by A and C. Q. E D.

The

The Leaver of the third kind hath its fix'd Point or Fulcrum at one End, the Weight at the other End as D at C, and the Power applied at
Fig. 25. any part between them, as at E B F, &c.

Now seeing that the Power applied must be always between the two Ends, therefore it follows that the Power must always exceed the Weight to be rais'd, or otherwise no Weight can be raised thereby.

Suppose the Leaver A C = 12 Foot A the Fulcrum, and at C is plac'd the Weight D = 50 Pounds. Now I say, That if the Power be
Fig. 25. applied in the middle at B, it must be = 100 Pounds Weight to equipoise D. For the Fulcrum being fix'd at A, it makes a Resistance equal to the Weight D, or rather a greater, or otherwise the Power at B could not raise it. Therefore,

As A B 6 the Distance of the Power from the Fulcrum,
Is to A C 12 the Distance of the Weight from the Fulcrum,
So is 50 the given Weight D to 100, the Power required at B to equipoise D at C.

Secondly, Suppose the Power to be applied at E 3 Feet from the Fulcrum A. Then I say,

As 3 Feet, the Distance of Power from the Fulcrum,
Is to 12 Feet the Distance of the Weight from the Fulcrum,
So is 50 the given Weight to 200, the Power required at E to equipoise D at C.

Again,

Suppose the Power to be applied at E 9 Feet from the Fulcrum A,

Then I say,

As 9 Feet the Distance of Power from the Fulcrum,
Is to 12 Feet the Distance of the Weight from the Fulcrum,
So is 50 the given Weight to 66 3 qrs. the Power required at E to equipoise D at C.

Now from these Examples 'tis also evident, that the further the Power is applied from the Fulcrum, the lesser the Power is required, altho' always greater than the Weight rais'd.

But however, altho' this kind of Leaver doth lose in its Power, contrary to both the others, yet it doth not lose in Time or Space also, as they do, but contrarily it gains in Space or Time proportionably:

As for Example,

Fig. 26. Let A E be a Leaver = 12 Foot, A the Fulcrum, E the Weight, and the Power applied in the middle at C.

Then I say, If the Power *c* raises the Leaver A E with the Weight E into the Position A B D, then will E have pass'd the Arch E D, which

which is = twice the Arch B *c*, through which the Power *c* hath mov'd.

For ſince that A *c* is = A E, therefore,

As A *h* is to *h* B, ſo is twice A *h* (that is, A *h* + *h* *g*) to *g* D, which is = to twice B *h*, the Triangles A B *h* and A D *g* being ſimilar. Q E D.

This kind of Leaver is chiefly us'd in the Regulators of Water Engines, where 'tis requir'd to ſtrike a greater Stroke than that of the Crank, as at *London Bridge*, where the Power of the crank Rods are applied between the forcing Rods and the Fulcrum of the Regulator.

This Leaver is alſo ſhewn by the raiſing of a Ladder, when the Power is applied in the Middle; the End reſting or kept down on the Ground as its Fulcrum, and the Weight beyond the Power, is the Weight requir'd to be rais'd.

A. But ſuppoſe that a Power is given with its Diſtance from the Fulcrum, as alſo the Length of the Leaver, pray how muſt I find out what Weight the given Power can equipoiſe?

B. By the following Canon or Analogy.

Suppoſe the given Power be 100 Pounds, applied at 7 Feet Diſtance from the Fulcrum, and that the Length of the Leaver is = 12 Foot.

ANALOGY.

As the Diſtance of the Weight from the Fulcrum,
Is to the Diſtance of the Power from the Fulcrum,
So is the Power applied to the Weight it will equipoiſe.

Then I ſay,

As 12 the Diſtance of the Weight from the Fulcrum is to 7 the Diſtance of the Power from the Fulcrum, *Fig.* 27.

So is 100 the Power applied at B to 58 3 qrs. the Weight at *c*. which it can equipoiſe.

OPERATION.

```
12   7   100
           7
         ———
   12)   700   (58  3 qrs.
         60
         ———
         100
         96
          4 = 1 qr.
```

These

Fig: 28. If a Leaver of the third kind as C A be continu'd beyond the Fulcrum = the Distance of the Power applied, (supposing B the Power given) that is, making E C = C B, then it will become a Leaver of the first kind, and the same Power which was applied at B, as a Leaver of the third kind, being applied at E as a Leaver of the first kind, will have the same Effect in equipoising the Body D. as when at B.

Suppose C A = 12 Feet, C B = 3 Feet, and let the Weight D sustain'd at A be = to 10 Pound. Then I say, that if A C be continu'd to E, making E C = C B 3 Feet, then the same Power requir'd to equipoise D at B being applied at E, will equipoise D also.

DEMONSTRATION.

First, Considering E A as a Leaver of the first kind, whose Fulcrum is C, and let the Weight D be = 10 Pound.

Then I say,

As E C 3 is to C A 12 :: So is D 10; to E 40; which is the Equipoise of D being applied at E.

Again,

Considering C A as a Leaver of the third kind, with the Power applied at B, 3 Feet from the Fulcrum C.

As C A 12; is to C B 3 : : So is, 40 applied at B to 10, its Equipoise at A.

Or, as C B 3, is to C A 12; so is 10 the Weight at A, to 40, the Power applied at B. Hence its plain, that the same Power has the the same Effect, either at E or B. Q E D.

These are the only distinct kinds of Leavers that are yet known, the fourth Leaver being no more than the Leaver of the first kind bended, or making an Angle at its Fulcrum; as the Handle of a Hammer, consider'd with its Head and Claws when us'd to draw a Nail, for then its Head is the Fulcrum, and it being between the Claws that lays hold of the Nail, and that part of the Handle where the Hand or Power is applied to draw the Nail, does therefore become a real Leaver of the first kind. And tho' 'tis call'd a Leaver of the fourth kind, or the bended Leaver, yet 'tis no more than a Leaver of the first kind, and the Analogies thereof are the same in all Respects.

N. B. That in the Practice of all these Operations, I have not made any allowance for the real Weights of the Leavers themselves, as at first noted, therefore always remember in Practice their own Weights must be allow'd for, exclusive of the Powers applied.

Having

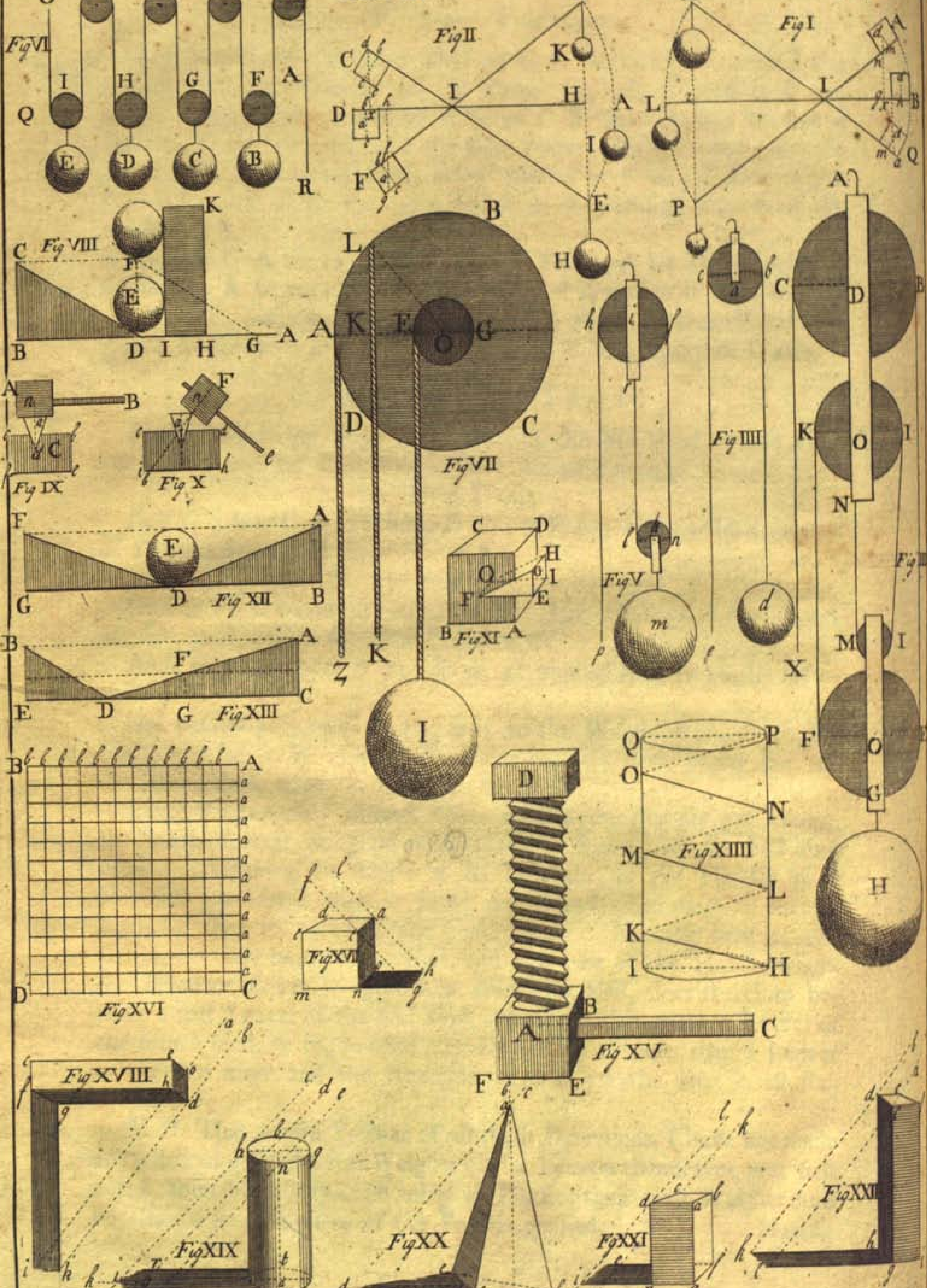
Fig I
Fig II
Fig III
Fig IIII
Fig V
Fig VI
Fig VII
Fig VIII
Fig IX
Fig X
Fig XI
Fig XII
Fig XIII
Fig XIIII
Fig XV
Fig XVI
Fig XVII
Fig XVIII
Fig XIX
Fig XX
Fig XXI
Fig XXII

Having thus demonſtrated the Powers of the Ballance, and ſeveral kinds of Leavers, by which Workmen may very readily know how to apply them in the beſt manner to practice in raiſing heavy Bodies; I ſhall in the the next Place proceed to ſpeak a Word or two relating to the Center of Gravity of Bodies, when ſuſpended at, or plac'd on, the End of the Leavers in horizontal and oblique Poſitions.

PROPOSITION I.

THEOREM.

If a Power as L, whoſe Line of Direction is perpendicular to the Leaver L I B, equipoiſe the Cube *d m*, whoſe Center of Gravity *d* is above the Leaver; its Power will be encreas'd as the ſaid Cube is rais'd above to A, and decreas'd as the ſaid Cube is let lower to Q, *&c.* Plate 3. *Fig.* 4.

A. Pray what is the Cauſe of the Powers Increaſe and Decreaſe, ſince it is not remov'd from the End of the Leaver.

B. The Cauſe is very plain; for by the Body's Change of the Place, the Line of Direction of both Power and Body are remov'd, whereby their Diſtances from the Fulcrum are not in the ſame Ratio as before.

1. Suppoſe the Cube *dm* on the Leaver B L to be remov'd to Q, then 'tis plain, that its natural Line of Deſcent or Direction *d m*, will become *d a*, wherefore the bearing of the whole Weight is at *a*, which is as much more further from the Fulcrum L, as the Line *a m*.

2. Suppoſe the Cube *d m* on the Leaver B L to be rais'd as far above B to A, as it was before depreſſed from B to Q, then will its Line of natural Direction or Deſcent *d m* become *d n*.

3. Since the Cube at A is at the ſame Diſtance from B, as Q is from B, therefore the other Ends K and P will alſo be equi-diſtant from L; and therefore if the right Line K P be drawn, it will cut the Leaver B L at right Angles in Z: Wherefore the Diſtance I z is = the Diſtance of the Powers in both the Leavers K Q and A P. But ſince, that by continuing the Lines of Direction *d a* and *d n*, up to the Leaver B L, they meet at unequal Diſtances from the Fulcrum in *x* and *g*, therefore the Cube at A, whoſe Diſtance *g* I is the leaſt from the Fulcrum I, requires leſs Weight than the Cube at Q, whoſe Diſtance *x* I is greater.

4. Now, ſeeing that a leſſer Power is requir'd at P than at K, 'tis evident that the ſame Power will equipoiſe a greater Weight, and

therefore the Power L in raising the Cube to A, is increas'd; and on the contrary as aforesaid, the Power L in letting down the Cube to Q is decreas'd. Q E D.

This is the first Variety of the Effects of raising Bodies. Now I will proceed to the second, wherein, by having the Center of Gravity plac'd below the Leaver, the Power is decreas'd when the Body is rais'd out of a horizontal Position, and increas'd when let fall below the horizontal Position.

This being the contrary to the preceeding, may perhaps at first Sight appear impossible. But observe,

1. That all Things being equal as before, the Distance of Power in the Leavers B F and E C are equal to H I; but the Distances of the Weights F and C are unequal, that of the Leaver B F being at *h*, and that of the Leaver E C being at *x*. Now, seeing that the Distance of the rais'd Weight C, which is the Point *x*, is farther from the Fulcrum I than the Distance of the let-fallen Body F, which is the Point *h*, and both the Distances of Power equal to one another, 'tis plain that the Power at K will be increas'd, and that at H decreas'd. Q E D.

Plate 3. *Fig.* 2.

Now, from the preceeding Rules, and those Remarks on the Manner of placing Bodies on a Leaver, it is impossible but that at all Times you may easily raise any heavy Body, with the least Power and in the least Time; which is the chief Work of Mechanicks.

A. Sir, I am very greatly oblig'd to you for the Favour of your kind Instruction of the Powers of the Ballance and Leaver, which I very truly comprehend; and therefore, beg Leave to ask the like Favour of the Pully, Windlace, Screw and Wedge. For, it is impossible that I can be a good and compleat Architect without knowing how to raise heavy Bodies by either of the several Powers, as Occasion may require.

B. 'Tis very true, a good Architect must be a good Mechanick, and therefore, in order to your Accomplishment, I will proceed to the Instruction thereof, which I will give you in the four following Lectures.

LECTURE VII.

Of the Power of the Pulley in raising of heavy Bodies.

A. WHAT is a Pulley?

B. A Pulley is a Wheel of Wood, Brass, Iron, &c. moveable about a small Pin or Axis, call'd the center Pin, to which in Theory

Theory we allow no Thickneſs, and therefore is conſider'd as a Line only.

Which ſaid Pin with the Wheel, is fix'd in a Box of Iron or Wood, &c. wherein it works.

A. How is a Pulley work'd?

B. By means of a Rope plac'd in the Grove of its Circumference, as you will preſently ſee in the following Practice.

A. Pray is there more than one kind of Pulley?

B. Yes, there are the ſingle Pulley (call'd by Workmen, a *ſnatch Block*,) and the double treble, &c. Pulley, call'd a pair of Blocks, &c.

A. Pray proceed to explain how heavy Bodies may be rais'd with the Power of the Pulley?

B. Obſerve, (1.) To equipoiſe the Weight by the ſingle Pulley *b a c*.

If the Diameter of the Pulley *b c*, be conſider'd as a Leaver of the firſt kind, wherein *a* is the Fulcrum, it is evident that *b* and *c*, the Extreams of the Diameter *b c*, are at equal Diſtances from the Fulcrum *a*; wherefore to equipoiſe the Body *d*, there muſt be a Weight at *e* equal thereto,

For, as *a c* the Diſtance of the Power, Fig. 4.
Is to *b a* the Diſtance of the Weight,
So is the Weight *d* to the Power *e*, or *Contra*,
So is the Power *e* to the Weight *d*.

Hence 'tis plain, that an upper Pulley, as *b a c*, is a Leaver of the firſt kind, and as its Fulcrum is at equal Diſtances from the Points of Diſtance of the Power apply'd and Weight to be rais'd, therefore the Power apply'd cannot equipoiſe any greater Weight than that which is = its ſelf.

A. Why then at this Rate, I don't ſee that the Pulley is of any Uſe, more than by its turning Motion it preſerves the Rope from fretting, and from a very great Friction, which would require an additional Strength when drew over an immoveable Body, as a Beam, &c. that would not turn as a Pulley doth?

B. Your Obſervation is right, but then it is in upper Pullies only: For in under Pullies, as the Pully *n k l*, 'tis otherwiſe.

A. Pray wherein is the Difference?

B. 1. If you obſerve, the Weight *m* hangs in the middle at *k*, and the Rope *f n*, is always lifting at *n*; and as the other Rope *l r*, is fix'd at *r*, therefore conſidering the Diameter of the Pulley *n k l*, as a

 Leaver

Leaver of the second kind, the Point *n* will be the Point where the Power is applied, and the Point *l* will be the Fulcrum. Then I say,

Fig. 5. As *k l*, the Distance of the Weight from the Fulcrum,
Is to *n l*, the Distance of the Power from the Fulcrum,
So is the Power applied at *n*, to the Weight that it will equipoise at *k*.

Hence 'tis plain, that as the Distance of the Power,
Is = twice the Distance of the Weight,
Therefore the Power apply'd will equipoise double its Weight; wherefore it is always to be understood, that by every such Pulley the Force is doubled.

Now from these two Examples arises the following

THEOREM.

When a Power (as X) sustains or draws a Weight by means of several Pullies, (as B C, I O K, L M, E O F;) each Pully under which the Rope goes, as E O F, or L M, is equivalent to a Leaver of the second kind, as before prov'd, and therefore needs no further Demonstration.

Fig. 3.

A. 'Tis very plain *Sir*, there needs no more to be said hereof: For 'tis evident, that every lower Pulley is a Leaver of the second kind; and as the Weight is always in the midst between the Power and the Fulcrum, 'tis very easy to judge or determine what Number of under Pullies are necessary to equipoise any Weight with a given Power.

As for Example,

Suppose a Body of 500 Pounds Weight is to be equipois'd by a Power of 25 Pounds Weight; How many under Pullies are requir'd for that Purpose?

This Question is easily answer'd: For as the Power is = double the Weight, therefore 25 Pound applied to one Pully, will equipoise 50 Pound.

Now if 500, the Weight given, be divided by 50, the Equipoise of one under Pulley, the Quotient will be 10, the Number of under Pullies requir'd. But if I am in an Error, pray *Sir*, shew me wherein?

B. Sir, you are not in an Error; your Observation is very just; which I am glad to find. And as you have so good an Understanding of the Nature and Power of the Pully, I will now proceed to shew you, that as much as the Power gains in Force, by means of many under Pullies, so much it loseth in Space and Time.

Suppose

Suppose a Power applied at A, which draws the Rope downwards to R to draw up or raise the four Weights B C D E of the Box P Q, on which they are fix'd.

Now I say, that in this and all such Cases, the Power A must descend or run thro' a great Space, whilst the Weights rise thro' a small Space; That is, the Power A must move eight Feet to raise the Bodies B C D E one Foot, because eight Parts of the Rope are applied to the lower Pullies.

Therefore observe in the use of Pullies, as in the use of Leavers,
That the Space which the Weight runs thro',
Is to the Space which the Power runs thro',
As the Power,
Is to the Weight, Or,
As the Number one,
Is to twice the Number of the lower Pullies, *viz.* 8.
So is the Power applied; to the Weight that it will equipoise.

Thus you see in the Pulley, as well as in the Leaver, this general Law is observ'd, That is to say,

That the more Velocity the Power has, the greater is its Force proportionably, which will also appear in the other following Powers: To which I proceed.

LECTURE VIII.

Of the Power of the Wheel by its Axel.

B. THIS Engine is of great Use at the several Keys and Wharfs of *London*, in raising and taking up all manner of Goods of Burden, at their loading and unloading into and out of Ships, Boats, &c. where the Power applied is the Weight of Men who walk within the Wheel, and thereby raise the Weights requir'd.

If you observe this Engine, and consider the Radius A O of the Wheel A B C D, with the Radius O G of the Axis, which move on their Center O, 'tis plain that it is nothing but a Leaver of the first kind perpetually turn'd round; for A O is the Distance of the Power, O G the Distance of the Weight, and the Center O the Fulcrum. And therefore, *Fig. 7.*

If a Weight is rais'd by means of such a Wheel, with its Axel moving round its Center, by a Power whose Line of Direction touches the

the Circumference of the ſaid Wheel; the Power will be to the Weight, as the Radius of the Axel, is to the Radius of the Wheel.

Suppoſe the Weight E, *Fig.* 1. or I *Fig.* 7. is rais'd by means of the Wheel, A B C D, with its Axel E O G, *Fig.* 2. moving round its Center O, by a Power Z, whoſe Line of Direction Z A touches the Circumference of the Wheel, as a Tangent rais'd from the Point A of the Radius A O; the Power A will be to the Weight I, as the Radius of the Wheel A O, is to the Radius O G of the Axel.

Let the Radius A O be = 10 Feet, the Radius O G = 1 Foot, and the Power applied at A = 15 Pound Averdupoiſe. Then I ſay,

Fig. 7. As the Radius of the Axel 1 Foot,
Is to the Radius of the Wheel 10 Feet,
So is 15 the Power applied to 150, the Weight at I, which is the Equipoiſe of the Power A required.

Again, The Weight given to find the Power,
As the Radius of the Wheel 10 Feet,
Is to the Radius of the Axel 1 Foot,
So is 150 the Weight given; to 15, the Power required.

OPERATIONS.

1 : 10 :: 15 : 150	10 : 1 : 150 : 15
10	1
1) 150 (150	10) 150 (15

From theſe Operations 'tis plain, that as much as the Radius of the Wheel is greater than the Radius of the Axel, ſo much is the Power of the Force increas'd, always ſuppoſing the Line of Direction of the Power to touch the Circumference of the Wheel, as A Z, whereby the Line O A Z will always be the ſame, and a right Angle, whatever Point of the Circumference it is applied: For were the Line of Direction otherwiſe applied, this would not hold. As for Inſtance.

Suppoſe the Power were applied at L, and its Line of Direction L K perpendicular to the Horrizon, then its evident, that the Diſtance of the Power from the Fulcrum would be but = K O; and ſince that K O is leſs than A O, 'tis plain that the Power is thereby diminiſh'd and made leſs, than when applied at A, as aforeſaid.

Having thus demonſtrated that the Power of the Wheel and Axel is gain'd by the Difference of their reſpective Radius, I muſt now proceed to inform you, that herein, as with the preceeding Engines,

whatever

whatever is gain'd in Force, is loſt in Time and Space. This is very eaſily underſtood; for as the Radius of the Axel make but one Revolution in the ſame Time that the Radius of the Wheel make one Revolution, 'tis evident that the Circumference of the Wheel, which is greater than the Circumference of the Axel, muſt move with greater Force, and that proportionably to the Difference of their Radius.

In this Example, the Cirumference of the Axel is $= 3\frac{3}{20}$ nearly, and the Circumference of the Wheel $= 31\frac{1}{2}$ nearly.

Now, If $31\frac{1}{2}$ be divided by $3\frac{3}{20}$, the Quotient is $= 10$. That is, the Circumference of the Axel is contain'd 10 Times in the Circumference of the Wheel; wherefore, the Wheel to raiſe the Weight one Foot in Height, muſt paſs thro' a Space of ten Feet; ſo that what it will have gain'd in Force, will be loſt in Space, according to the Difference of the Radius, which is, as 1 is to 10. Q. E. D.

LECTURE IX.

Of the Power of the Wedge.

B. SINCE the Power of this Engine is put in Action by Percuſſion or Striking, it is therefore to be firſt obſerv'd,

That the Center of Percuſſion, is that Point by which a Body, as a Beetle, &c. in its Motion ſtrikes with its greateſt Force another Body, (as a Wedge) which oppoſes its Motion.

That is the Point *a*, in the Middle of the Wedge *a d*, juſt under the Center of Gravity *n*, of the Beetle B A, is the Center of Percuſſion, and the Line *n a* its Line of Direction. *Fig.* 9.

And it is to be obſerv'd here, as before in the preceeding Engines, that the greateſt Force or Blow is given, when it falls perpendicularly upon the Wedge, and the Line of Direction of the Power as B A, parallel unto the Surface of the Wedge, on which the Stroke or Force is applied: For 'tis evident, that if the Stroke or Force is applied at oblique Angles, as the Beetle *e* F, the Line of Direction of the Force *n b*, will not be parallel to the centeral Line of the Wedge *a* I, nor will the Line of Direction of the Force *e n*, be parallel to the upper Surface of the Wedge, as in the former.

Now, ſeeing that the Line of Direction *n b*, is contrary to the Line of Direction *n a d*, 'tis evident, that the Beetle F *e* applied at oblique Angles, has leſs Force on the Wedge I, than when applid at right Angles, as the Beetle B A on the Wedge C.

A Sir,

A. Sir, I understand you perfectly well, pray proceed; but in the first Place, be pleas'd to inform me what is a Wedge?

B. A Wedge is the most plain or simple Engine that is, (call'd by some, tho' improperly, a solid Triangle;) it is either an irregular triangular Prism, as when its End or Height is perpendicular to its Base, as C I B, or an Isocles triangular Prism, when its two inclin'd Sides are equal to one another.

A. Of what Matter is the Wedge made?

B. The Wedge is made sometimes of hard Wood, as Box, &c. but generally of Iron, because that Iron may be made smoother, whereby it has less Stickage or Friction in cleaving Bodies, and therefore slides more freer against the Parts of the Body which it divides.

Now, to understand the Power of the Wedge, one of the inclining Sides is to be consider'd as a horizontal Plain, and we must imagine, that by the help of the inclin'd Surface, a Power shall raise a Weight which without this Machine could not so much as sustain or bear up.

Fig. 8. 1. Let the Triangle D B C, right angled at B, represent a Wedge, wherein let the angular Point D represent the Point or Edge, and the Perpendicular B C the Head, at which the Power is to be applied.

2. For the better understanding of the Power of this Engine, the the Lengths of its Base as D B, and Height B C must be given or known, that thereby their Proportions to one another may be known.

For the Analogy or Proportion is,

As the Height of the Wedge

Is to the Base of the Wedge,

So is the Power applied at its Center of Percussion, to the Weight that the Wedge will sustain or bear up.

Which will immediately be demonstrated; but 'tis to be noted, that in all Calculations of this kind, the Surfaces of the horizontal Plain, whereon the Wedge slides, the Surfaces of the Wedge, the Surface of the Body to be rais'd, are suppos'd to be so very smooth, as to slide without Obstruction or Difficulty.

Again, It must also be suppos'd, that the Weight E be hindred from going to A, by the perpendicular Plain H I K; which however is to be suppos'd not to hinder the Wedge D C B from sliding along the horizontal Plain A B, as 'tis driven or forc'd from B towards the Power applied at B, whose Line of Direction is parallel to the Horizon.

Now,

Now if the Power applied at B drives or forces the Wedge C D B regularly from B towards A, upon the horizontal Plain A B, it will cause the Weight E to ascend by so regular a Motion, that its Center of Gravity E, will by its Ascension generate the right Line F D perpendicular unto the Horizon; so that when the Point B shall be come to D, the Point C to E, and the Point D to G, the whole Wedge C D B shall have chang'd its Place to E D G, and the Body E will be rais'd the whole Height of C B, the perpendicular Height of the Wedge.

Now if B B the Base be = twice B C the Height, then the Power shall have mov'd the Line D B its whole Length, which is = twice C B, and therefore the Power to the Weight will be as 1 is to 2. That is,

As the Height C B one,

Is to the Base D B two,

So is the Power of one Pound applied by Percussion at B, to two Pound the Weight of the Body E rais'd to F.

From hence it follows, that the more acute the Wedge is, the greater will its Effect be; because G D, the Velocity of the Power will be great in Comparison of D F, the Velocity of the Weight. *Fig.* 11.

When the Wedge is applied to cleave a Body, as A B C D, the Plains E F O I, and G F O H, which make up the Wedge, being more inclin'd to each other, the Parts E G will therefore slide more easily.

If the Plain E F O I be consider'd as a horizontal Plain, the other Plain G F O H will be an inclining Plain; wherefore the Resistance of the upper Part of the Body A B D C, which is to be disunited from the lower, may be look'd upon as a Weight, whose Line of Direction is perpendicular to the lower, or horizontal Part, and then a Power applied as aforesaid, with an additional Power for the Roughness of the Body, when irregular, will have the Effect desir'd.

You must also take Notice, that altho' the Power of a Blow or Stroke is = a certain Weight, yet if that Weight is laid gently upon a Wedge, it will not have the same Effect in forcing it into a hard Body, as when the same Weight is communicated to it by a Blow.

A. That is very strange, since that the real Weight gently laid on is the same as that communicated by the Blow. Pray what is the Cause of it?

B. None can as yet assign the real Cause thereof; but my Opinion is, that a Blow by putting all the Parts into Motion, makes them to tremble, and so disunite the Particles of its Surfaces from the Particles of the Body next to them, which before had a close adherence, wherefore the Wedge more freely enters.

And it is to be also obſerv'd, that the Effect of Percuſſion will be greater in Proportion, as the Percutient or ſtriking Body is heavier and ſwifter: That is to ſay, the heavier the Body is with which you ſtrike, as a ſledge Hammer, *&c.* and the greater Velocity you ſtrike with, the greater Effect it will have.

Therefore,

If with a Body of 10 Pound Weight, a Stroke be made in one Second of Time, that will raiſe 20 Pound Weight, the ſame Body being ſtruck in like manner, with double the Force, that is, in half a Second of Time, it will raiſe 40 Pound Weight, which is double to the former.

Hence it appears, that the Power of Percuſſion is proportionable to the Velocity of the Stroke.

A. It ſeems very plain: But I can't ſee of what great Uſe the Wedge is in raiſing of Bodies of Weight, ſince that there muſt be a reſiſting Plain as K H I, which in many Caſes cannot be practicable, and therefore I think that the Wedge is of little Uſe.

Fig. 12. *B.* You are miſtaken *Sir*, any heavy Body may be rais'd without a plain perpendicular to reſiſt it, as follows.

Suppoſe the Body E is to be rais'd to the Height of the Wedge F G, 'tis plain that if againſt the Wedge F D G, you place another Wedge, as A B D equal thereto, ſo as to work cloſe to each others Sides, and each Wedge being driven with equal Force, will raiſe the Body E the Height requir'd. And as I ſaid before, the longer or more acute the Angle of the Wedge is, the eaſier the Weight will be rais'd. This has been already prov'd, and therefore its needleſs to repeat it again.

Now I muſt conclude this Lecture with the ſame Obſervation as in the former Lectures.

That as much as is gain'd in Force by the Acuteneſs of the Wedge, ſo much is loſt in Space and Time, becauſe that the more acute a Wedge be made, the greater Length it muſt be, to be equal in Height to another Wedge, whoſe Angle is leſs acute, or rather, whoſe Angle contains a greater Number of Degrees.

That is, the Wedge A C D, whoſe Line A D C is leſs than the Line B D E, muſt be longer than the Wedge B D E, to be equal in Height thereto, for was the Wedge A D C to be no longer than the Fig. 13. Wedge B D E, that is, G F D E, then it could not raiſe the Body higher than F. Hence 'tis evident, that D F muſt be continu'd to A, and D G to C, whereby it will raiſe the Body to the Height requir'd. And ſince that tho' the Wedge A D C, will move with leſs

Force

Force than the Wedge B D E, yet it requires more Time, becauſe its Length C D is greater than D E. Therefore 'tis plain, that what is got in Force is loſt in Space, as has been already prov'd in the other Engines.

LECTURE X.

Of the Power of the Screw.

A. WHAT kind of an Engine is the Screw?

B. The Screw is a Cylinder, cut into ſeveral concave Surfaces continually inclin'd, or in plainer Terms, it is the Wedge winded about the Convexity of a Cylinder, with a certain and equal Inclination, whoſe each Circumvolution is call'd a Helix or Thread of the Screw, as P O, N M L K H.

This Engine is very Uſeful, as I ſhall hereafter ſhew, to move or preſs with great Force. Fig 14.

It was from the right angled Triangle, or inclin'd Plain, that the firſt Hint was given to the Inventers of the Wedge, and it was from the Wedge that the firſt Hint was given to the Inventers of the Screw, which was made by the winding the ſaid Triangle about the Convexity of a Cylinder, as the Triangle H K I, about the Cylinder H I P Q; whereby it became of more Uſe, and was contain'd in leſs Space: For which End the Height of the Triangle has been allow'd for I K the Height of the Cylinder, and the Inclination of the Hypothenuſe of the ſaid Triangle has been given to the Helix or Thread H K, and ſo in like manner to all the other Helixes that go upwards round about the Cylinder of the Screw, which in Fact doth make the Thread or Helix an actual ſpiral Line, wound about the Convexity of the Cylinder.

Since that the Screw is no other than the Wedge, it therefore follows, that if a Power ſuſtain a Weight by means of a Screw, that Power will be to that Weight, as the Height of the Screw is to the Thread of the Screw.

That is, if the whole Line or Thread of the Screw was unwound from the Convexity of the Cylinder, and laid at full Length,

Then the Power applied,

Will be to the Weight that it will equipoiſe,

As the Height of the Cylinder is to the Length of the extended Thread.

Whence it is easy to conclude, that in a Screw the Force of the Power is the greater, the nearer the Circumvolutions of the Thread are together, and the more they are inclin'd to the Horizon; because then the Height of the Cylinder is capable of containing a greater Length of the Helix or Thread, and consequently the Helix will have a greater Ratio to the Height of the Cylinder, whereby the Power will likewise have a greater Ratio to the Weight to be rais'd.

But to make an Estimate of the Force of a Screw, we have no Occasion to measure the whole Length of the Thread, nor the Height of the whole Cylinder; for if we know how often the Height of one Thread from the other is contain'd in one Circumvolution or Helix, that is, how often the Height H L is contain'd in the Circuit of the Helix H K L, because H P, the whole Height of the Cylinder, is contain'd just as many times in the whole Thread of the Screw H K L M N O P, and therefore the Force is the same.

Hence 'tis plain, that the Screw can raise a Weight by one Helix or Thread, no higher than from H to L, and that if the Height H L, is contain'd ten times in the Helix H K L, a heavy Body will by means of this Engine be sustain'd by a Power little more than = one tenth Part of its Weight.

This Engine is always work'd with a Leaver of the second kind, as C A, whose Fulcrum is the Center of the Cylinder A; Distance of Weight = the Radius of the Cylinder and Power at C, *&c.* and as it has been before prov'd, that the farther the Power is applied from the Fulcrum of a Leaver, the greater is its Force, so 'tis plain that by encreasing the Length of the Leaver, the Force may be also increas'd at Pleasure; but then what is here gain'd in Force will be lost in Time and Space, as has been already prov'd in all the preceeding Engines.

LECTURE XI.

Of the Mensuration of Superfices and Solids, and first of Superfices.

A. WHAT is a Superfices?

B. A Superfices is a Space bounded by one or more Lines, consisting of Length and Breadth, but of no Thickness, as the Circle
Plate 1. *a b c e. Fig,* 2. bounded by one Line, call'd the Circumference as aforesaid. The Triangles *a b c d, Fig.* 4. The Square *a b c d, Fig.* 6.

Fig. 6. The Oblong *a b c d*, *Fig.* 7. The Rhombus, *Fig.* 8. and the Trapezium, *Fig.* 11. by four Lines. The Octagon D by eight Lines. The irregular Surface, *Fig.* 13. by many Lines or Sides, &c.

A. Pray how are these several Figures to be measur'd?

B. By the Knowledge of their Lengths and Breadths being given in any kind of Measure, as in Inches, Feet, Yards, &c. As for Example, Suppose each Side of the Square F was equal to 12 Feet, then the Content is easily found by multiplying 12 into 12, that is, its Length into its Breath, and Product will be = 144, which I thus demonstrate.

Suppose each Side of the Square A B C D be 12 Feet, divided into 12 equal Parts at the Points *a a*, &c. and *b b b*, &c. Then I say, if the right Line *a a a*, &c. be drawn parallel to the Side A B, and the Lines *b b*, &c. parallel to the Sides A C, they will constitute 144 little Squares each = one square or superficial Foot, the Product of 12 the Length multiplied into 12 the Breadth. Q. E. D. Fig: 16. Plate 3.

A. I see the Reason of Multiplication very plainly, and since that many Examples will be very troublesome, I desire that you will only inform me of the Canons or Analogies by which every Figure is measur'd without any further Demonstration, which at present I have not Time to consider, and therefore should be glad if you will immediately proceed to practice in such a manner as you think is best for my Instruction.

B. I will, and

First, to measure any right lin'd Triangle, as d n o, &c. *Fig.* 5. Plate 1.

RULE.

1. Let fall a perpendicular Line from the angular Point opposite to the Base, upon the Base, as *n g*, then multiplying half G the Length of the perpendicular Line, by the whole Length of the Base *d o*, the Product is = the Content, or multiply the whole Length of the Perpendicular by the half of the Base, and the Product is the Content required, or multiply the whole Base by the whole Perpendicular, and half the Product will be the Content of the Triangle requir'd.

Secondly, To measure any Geometrical Square.

RULE.

Multiply any one Side into its self, and the Product is the Content required.

Thirdly,

Thirdly, To measure an Oblong or Parallelogram.

RULE.

Plate 1. Fig. 7. Multiply the Length *a b*, into its Breadth *a c*, and the Product will be the Content required.

Fourthly, To measure a Rhombus.

RULE.

Multiply the Diagonal *e d* by the Diagonal *b e*, and half the Product will be equal to the Content required. Fig. 8.

Fifthly, To measure a Rhomboyades.

RULE.

1. Let fall a Perpendicular, as *a z*, or *d g*, then multiply the Length *a b*, or *c d*, by the Length of the Perpendicular *a z* or *g d*, and the Product will be the Content required. Fig. 9.

Sixthly, To measure a Trapezium.

RULE.

Divide the Trapezium into two Triangles, as *a b c*, and *b c d*, then letting fall the Perpendiculars *a x*, and *e b x*, measure the Content of each Triangle separately, and afterwards adding their Contents together, the Sum will be the Content of the Trapezium required. Fig. 11.

Seventhly, To measure an irregular Figure, as a b c d e f g h.

RULE.

Fig. 13. 1. Divide the given Figure into Triangles, and measure each Triangle separately.

2. Add together into one Sum all the Contents of the several Triangles, and the Sum total will be the Content required.

Plate 1. Fig. 12. *Eightly, To measure a Polygon, as a Pentagon, Hexagon, &c. Suppose a Hexagon.*

RULE.

1. Let fall a Perpendicular from the Center A to one of its Sides, as *a b*, then multiplying half the Circumference by the Perpendicular, the Product will be the Content required.

Ninthly, To measure a Circle.

RULE.

Square the Diameter given (which is, to multiply it into its self) and its Product multiply by 11, and the Product last produced being divided by 14, the Quotient is the Content required.

Or

Or thus,

Multiply half the Diameter given by half the Circumference, and the Product will be the Content required.

A. But how must I know the Circumference of a Circle when I do not know it, having the Diameter only given?

B. As follows. The Proportion (in Practice to be observ'd) that the Diameter of a Circle hath to its Circumference, is, as 7 is to 22; and so is the Diameter given, to the Circumference required. That is, suppose the given Diameter 12 Feet. Then I say,

As 7: is to 22:: So is 12: to 37 $\frac{5}{7}$

22
———
24
241
———
7) 264 (37 $\frac{5}{7}$
21
———
54
49
———
05

And so contrarily

If the Circumference was given to find the Diameter,

Then the Analogy is,

As 22: is to 7: So is the Circumference given; to the Diameter required.

Suppose the given Circumference be 125 Feet,

Then

As 22 : 7 :: 125 : 39 $\frac{17}{22}$ the Diameter required.

7

22) 875 (39 $\frac{17}{22}$
66
———
215
198
———
17

Tenthly, To measure an Ellipsis.

1. Find a mean Proportional between the two Diameters, and supposing

posing the mean Proportional to be the Diameter of a given Circle; the Area thereof is equal to the Area of the Ellipsis required.

These are the general Rules for the Mensuration of Geometrical Superfices, by which all superficial Works may be easily measur'd, either in Inch, Feet, or Yard Measure; and therefore I shall in the next Place proceed to the Mensuration of Solids, and therein, as well of their superficial Quantities as their cubical or solid Contents.

LECTURE XII.

Of the Mensuration of solid Bodies.

B. THE Solids that I shall here shew the Mensuration of, are such that occur in the Practice of Building, and

First, To measure a Cube.

RULE 1. For the solid Content.

Multiply the Area of one of its Surfaces by its Depth or Thickness, and the Product is the solid Content required.

RULE 2. For the superficial Content.

Multiply the Area of one of its Surfaces by 6, the Number of its Surfaces, and the Product is the superficial Content required.

Secondly, To measure a Parallelopipedon, or long Cube.

RULE 1. For the solid Content.

Multiply the Area of any one Surface thereof, by the Length contain'd between that Surface and the Surface opposite thereto, the Product is the solid Content.

RULE 2. For the superficial Content.

If the Breadth is = the Depth, multiply the Girt by the Length, and to their Product add the Area of each End, and their Sum shall be = the superficial Content required.

But if the Breadth is not = the Depth, then find the Area of one End, one Side, and the Base or Top, and their Areas being doubled and added together, will be = the superficial Content required.

Thirdly,

Thirdly, To measure a Sphere or Globe.

RULE 1. For the superficial Content.

Multiply the Area of a Circle, whose Diameter is = the Diameter of the given Sphere by 4, and the Product is the superficial Content required.

RULE 2.

Multiply the Circumference by the Diameter, and the Product will be nearly equal to the superficial Content required.

RULE 3. For the solid Content.

Multiply the Area of the Sphere by one third Part of its Radius, or one sixth Part of its Diameter, and the Product will be the Content required.

Fourthly, To measure a Cylinder.

RULE 1. For the superficial Content.

Multiply the Girt by the Length, and to the Product add the Areas of each End, the Sum is the superficial Content required.

RULE 2. For the solid Content.

Multiply the Area of one End by the Length, and the Product is the Content required.

Fifthly, To measure a Cone.

RULE 1. For the superficial Content.

Multiply the Side of the Cone, by half the Circumference of the Base, to that Product add the Area of the Base, their Sum will be the superficial Content required.

RULE 2. For the solid Content.

Multiply the Area of the Base by one third Part of the Axis, or perpendicular Height of the Cone, and the Product will be the solid Content required.

Sixthly, To measure the Frustum of a Cone.

RULE 1. For the superficial Content.

First, Add into one Sum the Circumferences of the greater and the lesser Base, which being multiply'd by half the Length of the Side (not the Axis) and to the Product add the Area of each Base, their Sum will be the superficial Content required.

RULE 2. For the solid Content.

First, Find the Area of each Base, and add them together in one Sum, which note by its self.

Secondly, Multiply the Areas of the two Bases into one another, and extract the square Root of their Product

Thirdly, Add the square Root of the Product to the Sum of the two Areas first found, into one Sum, which being multiply'd by one Third of the Frustum's Length, the Product is the Content required.

Seventhly, To measure a Pyramid.

RULE 1. For the superficial Content.

Multiply half the Circumference of the Base by the Length of the Side of the Pyramid, and to that Product add the Area of the Base, and their Sum is the superficial Content required.

RULE 2. For the solid Content.

Multiply the Area of the Base by one third Part of the Length of its Axis, or perpendicular Height, and the Product will be the solid Content required.

Eighthly, To measure the Frustum of a Pyramid.

RULE 1. For its superficial Content.

Measure each Surface as before taught of the Trapezium, and their Sums added together will be the superficial Content required.

RULE 2. For the solid Content.

First, Add into one Sum the Area of each Base.

Secondly, Multiply the Area of the greater Base by the Area of the lesser Base, and extract the square Root of their Product.

Thirdly, Add the square Root of their Product to the Sum of the two Areas first found together, and their Product being multiply'd by one third Part of the Length of the Frustum, the Product will be the solid Content required.

Thus have I in a plain Manner, and in the fewest Words, given you Rules for the Mensuration of all manner of Geometrical Superfices and Solids, by Help of which you may with great Exactness measure all manner of Works as they occur in Practice.

Of

Of the Menſuration of Artificers Work by the ſliding Rule, Commonly call'd Coggleſhall's *Rule.*

LECTURE I.

B. 1. THE Rule which I am now going to teach the Uſe of, is generally plac'd upon one Side of one Leg of a two Foot Rule, conſiſting of four Lines. Of which, two Lines are plac'd on a Slip or ſliding Part, and the other two upon the Leg of the Rule, and are thereby fix'd. Theſe laſt two Lines which are plac'd on the Rule, I ſhall hereafter in Practice call the *Stock*, and the two middle Lines the Slip.

2. You ſee by the Figure hereunto annex'd, that the uppermoſt Line on the Stock, and the two Lines on the Slip, are all alike divided and number'd, *viz.* from 1 2 3, *&c.* to 1 in the middle, and from thence to 10 at the End, and the undermoſt Line from 4, 5, 6, *&c.* to 40, which Line is call'd the ſquare Line; and when us'd in Timber, the Girt-line.

3. Obſerve alſo, that the Lines on the Slip, and that next above it, are each divided between 1 and 2 into 10 Parts, and each Tenth ſubdivided into 5 Parts, and conſequently the whole Diviſion contain'd between 1 and 2, is thereby divided into 50 Parts. And if you ſuppoſe each Diviſion to be again ſubdivided or equal to 2; then the whole Space contain'd between 1 and 2, may be ſaid to be divided into 100 Parts.

4. The Spaces between 2 and 3, and 3 and 4 are each decimally divided into ten Parts as before; and as the Diſtance between 2 and 3, is leſs than between 1 and 2, therefore theſe Tenths are each ſubdivided into two Parts, and conſequently the whole Space between 2 and 3, will be divided into but 20 Parts; and if each Part is accounted to be divided into 5 leſſer Parts, or each = 5, then the whole Space between 2 and 3, may be or ſuppoſed to be divided into 100 Parts as before between 1 and 2.

5. The other Diviſions between 4 and 5; 5 and 6; 6 and 7; 7 and 8; 9 and 9; being yet leſſer and leſſer, are therefore each divided into ten Parts only, wherefore accounting each Part equal to ten, then

every of those Divisions may be said or supposed to be divided into 100 Parts, as the preceeding.

6. The remaining Length from 1 in the middle to 10 at the End, is divided respectively in the like manner, as also are the respective Divisions of the Girt-line on the Stock, which are contain'd between the beginning thereof at 4 to 10: But the Divisions from 10 to 40 are first each decimally divided into 10ths, and each 10th subdivided into 4 Parts.

A. I understand you perfectly well, but I observe, that on the Girt-line, just at the beginning before 4 there are two Divisions, each subdivided into four Parts; and at the End beyond 40, there are two Divisions, each subdivided into two Parts; Pray what do they signify, for I observe that you have not taken Notice of them.

B. The two Divisions at the End of the Line beyond 40, each subdivided into two Parts, are the same as the first two Divisions subdivided into Halves, that are next after 4 at the beginning of the Line; that is, if you suppose that from 40 the whole Line was immediately to begin again, placing 4 the Beginning in the Place of 40 at the End, then those two Divisions would represent the first two Divisions between 4 and 5.

Likewise the two Divisions subdivided into four Parts, placed before 4 at the Beginning, are equal to the two last Divisions next before 40 at the End.

That is, supposing that the Space between 30 and 40 at the End of the Line was to be prefix'd before 4 at the Beginning, then would the two last Divisions before 40 be in the same Place of the two Divisions before 4.

A. Sir, I understand you; pray proceed to shew me how to read or express Quantities by these Lines.

B. To numerate or express Quantities on this Rule, observe,

1. Let the Space between 1 at the Beginning of the Line, and 1 in the Middle, represent one Integer, as one Foot or one Inch, *&c.* then will 1 at the Beginning signify one Tenth thereof; 2 will signify two Tenths; 3 will signify three Tenths; 4 four Tenths; 5 five Tenths; 6 six Tenths; 7 seven Tenths; 8 eight Tenths; 9 nine Tenths; and lastly the 1 in the Middle 1 Integer as aforesaid: And as before was shewn, that every such principal Division of 1, 2, 3, 4, 5, 6, 7, 8, 9; 1 was severally divided into 100 Parts, therefore this Integer is thereby divided into a thousand Parts.

2. The

2. The following Divisions from 1 in the Middle, as 2, 3, 4, 5, 6, 7, 8, 9, 10, are severally whole Integers; that is, when the Space from 1 at the Beginning to 1 in the Middle is reckon'd the Integer, then the 2 following signifies 2 Integers; the 3 signifies 3 Integers; the 4, four Integers, &c. and so consequently the End of the Line signifies 10 Integers, and their respective Sub-divisions, represents their fractional Parts, as before shewn in the first Integers.

3. But if 1 at the Beginning of the Line be accounted an Integer, or one, then the 2 following signifies two Integers; the 3 following signifies three Integers; the 4 following signifies four Integers, &c. and the 1 in the Middle signifies ten Integers. Now as the 2 following the 1 in the Middle did before represent two, when the 1 in the Middle represented one Integer, so now will the same 2 represent twenty, when the 1 in the Middle represents ten Integers, and consequently the following Numbers 3 4 5 6 7 8 9 10, will represent Three Hundred, Four Hundred, Five Hundred, Six Hundred, Seven Hundred, Eight Hundred, Nine Hundred, and lastly a Thousand; and so in like manner if the Line be began with 100, then the 1 in the Middle will represent 1000, and the 10 at the End 10000, &c.

4. When the Line is began with 1, and the middle 1 signifies 10, then every Decimal or tenth Division following between 1 and 2, between 2 and 3, &c. will represent an Integer. Thus the first tenth Division after the 1 in the Middle signifies Eleven: the second tenth Twelve; which is number'd 12, with a smaller Figure than the others, and sometimes only distinguish'd by four Points, thus ∴, and in like manner all others, of which every fifth is distinguish'd by a longer Stroke than the others, as 15, 25, 35, &c. and the Sub-divisions of each Integer are fractional Parts thereof.

5. When the Line is began with 10, then every tenth Division between 1 and 2, between 2 and 3, &c. doth each represent an Integer (as before was said of the Tenths following in the Middle of the Line.) Thus the first tenth after 1 at the Beginning signifies 11, the second tenth 12; the third tenth 13; the fourth tenth 14; the fifth tenth 15, (which hath its Division longer than the others as aforesaid,) the sixth tenth 16, &c.

A. Sir, I understand the Numeration of this Line very well; pray proceed to its Use in Mensuration.

B. I will. But *First*, I must shew you how to multiply and divide one Number by another, as also how to perform the Rule of Three, and extract the

the ſquare Root, after which your Menſuration will be very eaſy and delightful, wherein obſerve,

Firſt, For Multiplication.

The Analogy is

As 1 is to the Multiplyer: : So is the Multiplicand to the Product.

EXAMPLE 1.

Multiply 7 by 9.

Practice, Begin the Line with 1, and ſet 1 on the Slip to 7 in the upper Line of the Stock, and againſt 9 on the Slip ſtands 63 on the upper Line of the Stock, which is the Product required.

EXAMPLE 2.

Multiply 10 by 12.

Practice, Begin the Line with 10; Set 1 on the Slip to 10, on the Stock, and againſt 12 on the Slip ſtands 120 on the Stock, which is the Product required, and ſo in like manner any other given Numbers.

Secondly, For Diviſion.

The Analogy is

As the Diviſor is to 1: : So is the Dividend to the Quotient required.

EXAMPLE 1.

Divide 72 by 9,

Practice, Begin the Line with 1, place the Diviſor 9 on the Stock, againſt 1 on the Slip, and againſt 72 on the Stock, ſtands 8 on the Slip, which is the Quotient required.

EXAMPLE 2.

Divide 630 by 15.

Practice, Begin the Line with 10, then againſt the Diviſor 15 on the upper Line of the Stock ſet 1 on the Slip, and againſt 630 on the the Stock ſtands 42 on the Slip, which the is Quotient required.

Thirdly, For the Rule of Three.

The Analogy is

As the firſt given Number is to any other Number (as 5 is to 11, &c.) ſo is the ſecond given Number (as 10) to a fourth Number, which is the Number ſought for, in the ſame Proportion.

EXAMPLE

EXAMPLE 1.

If five Men are paid 11 Pounds for one Weeks Work, what muſt ten Men be paid for the ſame Time, having the ſame Allowance?

Practice, Begin the Line with 1, then ſet 5 on the Slip againſt 11 on the Stock; and againſt 10 on the Slip ſtands 22 on the Stock, the Number of Pounds required for the Payment of ten Men in the ſame Proportion.

EXAMPLE 2.

If the Diameter of a Circle be 7 Feet whoſe Circumference is 22 Feet; what is the Circumference of another Circle whoſe Diameter is 22 Feet?

Practice, Begin the Line with 1, then ſet 7 on the Slip againſt 22 on the Stock, and againſt 21 on the Slip ſtands *66*, the Circumference required.

EXAMPLE 3.

If 21 Bricks pave 1 ſquare Yard, How many Bricks will pave 30 Yards?

The Analogy is as 1 is to 21, ſo is 30 to the Number required.

Practice, Set 1 on the Slip to 21 on the Stock, againſt 30 on the Slip ſtands 630, the Number requir'd for the Pavement of 30 Yards.

Now in working of the Rule of Three Direct, as in the preceeding Examples, you ſee, that as the ſecond N°. is always greater than the firſt; the fourth N°. will be alſo greater than the third; *et contra*.

And in the Inverſe Rule as the Example following; If the ſecond be leſs than the firſt, the fourth ſhall be leſs than the third, *et contra.*

EXAMPLE 4.

If the Circumference of a Circle be 44 Feet, whoſe Diameter is 14 Feet, what will the Diameter of another Circle be, whoſe Circumference is *66* Feet.

The Analogy is, as 44 is to 14 ∴ So is *66* to the Number required.

Practice, Begin the Line with 1, and againſt 44 on the Stock, ſet 14 on the Slip, then againſt *66* on the Stock ſtands 22 on the Slip, which is the Diameter required.

Now from theſe Examples 'tis plain, that the ſecond and third Numbers are never taken on the ſame Line, which you are always to remember.

Alſo

Alſo obſerve,

If placing the firſt Number to the ſecond, the third falls beyond the Line, take the third Number in the firſt Part, or the other Length of the Line as if it was continu'd; giving it its Value according to its Place as before ſhewn.

Fourthly, Of the Extraction of the ſquare Root.

B. By help of the lowermoſt Line on the Stock before call'd the ſquare Line, or girt Line, the ſquare Root of any Number not exceeding ten Thouſand, may be very readily found, as following.

Practice, Begin the lowermoſt Line of the Slip with 10, and ſet 16 thereof to 4, the beginning of the ſquare Line, then the Numbers of the ſquare Line will be the ſquare Roots of the Numbers contain'd in the lower Line of the Slip, or thoſe Numbers in the Slip will be the Squares of thoſe of the girt Line on the Stock. Thus againſt 5 on the girt or ſquare Line, ſtands 25 on the Slip, and againſt 6 on the girt Line ſtands 36 on the Slip: So in like manner.

Againſt	7 8 9 10 20 30	ſtands	49 64 81 100 400 900	which are the ſquare Numbers of	7 8 9 10 20 30

Hence 'tis plain, that any given ſquare Number under 1000 being found in the lower Line of the Slip, its ſquare Root or Side of its Square is that Number in the ſquare or girt Line which is oppoſite thereto.

2. Remove the Slip, and place its beginning 1 to 10 on the ſquare or girt Line, and accounting 1 the beginning of the Slip, as 100, then 40 on the ſquare or girt Line ſtand againſt 1600, its Square in the Slip; and thus you have the Root of any ſquare Number under 1600.

3. Remove the Slip to its firſt Station, placeing 16 on the Slip (beginning the Line with 10) againſt 4, the beginning of the ſquare Line; and then reck'oning the ſaid 16 in the Slip to be 1600, and the 4 in the girt Line againſt it to be 40; as when thoſe Numbers were together at the other End in their laſt Station; then will the ſquare Num-

Numbers in the Slip go on from 1600 to 10000, whose Roots are contain'd in the square Line opposite thereto. Thus,

The square Root of	1600	is	40	as exhibited by the opposite Divisions in the square or girt Line.
	2500		50	
	3600		60	
	4900		70	
	6400		80	
	8100		90	
	10000		100	
	40000		200	
	90000		300 &c.	

And removing the Slip, as at the second Operation you may continue the square Numbers to 160000; and then altering the Slip as at the third Operation you may continue them from 160000 to 10000000; and so on in like manner *ad Infinitum.*

I shall now proceed to Artificers Works, and

First, Of Masons Work.

A. Pray how are the Dimensions of Masons Works measur'd?

B. With Feet and Inches, wherein there are two kinds, that is, the one superficial Measure, by which all manner of Pavements, Chimney Pieces, Cornishes, &c. are measur'd, and the other solid, by which Columns and other massy Parts of Buildings are measur'd, and therefore the Price of Masons Work is either at *per* Foot, superficial or solid, excepting where running Measure is agreed on.

The Analogy for Foot Measure is the following.

As 12 on the upper Line of the Stock, which in Foot Measure is always fixt, and therefore noted with small Figures as before noted,

Is to the Dimensions Length in Feet and Parts of Feet accounted on the Slip,

So is the Breadth in Inches accounted on the upper Line of the Stock,

To its Content in Feet on the Slip.

EXAMPLE.

A Piece of Marble Pavement is 36 Foot and half in Length, and 33 in Breadth, what's the superficial Content?

Practice, Set the Length 36 $\frac{1}{2}$ Feet on the Slip to 12 on the upper Line of the Stock, and against 33 on the Stock stands 100 $\frac{1}{3}$ the Content required.

And here note, that when Fractions happen as in this Example, they are to be estimated as near to the Truth as can be, which in Practice of Business is near enough for our Purpose. But to determine the true Quantity or Value of fractional Quantities (it being impossible to be done by this Rule) you must have Recourse to vulgar or decimal Arithmetick, with which it is hop'd you are already well acquainted.

EXAMPLE 2.

There is a *Portland* Slab, 8 Feet 3 Inches in Length, and 17 $\frac{1}{2}$ Inches in Breadth, what's the Content?

Practice, Set the Length 8 Feet 3 Inches on the Slip, (which is 8, and two and half of the Sub-divisions of the Tenths) to 12 on the upper Line of the Stock; and against 17 Inches $\frac{1}{2}$ on the Stock stands 12 and a very little more, which is equal to 4 $\frac{1}{2}$ square Inches, the true Content required.

And so in like manner, any other Quantities as given.

The next in Order is solid Measure, which Business generally happens under the Forms of the Cylinder, the Cube and the Parallelopipedon.

First, Of the Cylinder.

The Analogy is the following.

As the Length in Feet and Inches accounted in the lower Line of the Slip,

Is to 10, 635 accounted on the girt Line,

So is $\frac{1}{4}$ of the Circumference or Girt in Inches,

To the solid Content in Feet as required.

EXAMPLE.

There is a Cylinder of Stone, whose Length is 28 Feet 9 Inches; and $\frac{1}{4}$ of its Girt or Circumference 15 Inches, what's the solid Content?

Practice, Set the Length 28 Feet $\frac{3}{4}$ in the Slip, to 10 635 in the girt Line, and against 15 the Quarter of the Cylinder's Girt, accounted on the Girt Line, stands 61 on the Slip, the solid Content required.

Secondly,

Secondly, Of the Cube.

The Analogy is the following.

As the Length or Side of the Cube in Feet and Inches accounted in the lower Line of the Slip,

Is to 12 accounted in the girt Line,

So is the Depth or Side of the Cube in Inches, accounted in the girt Line, to the ſolid Content in Feet accounted in the Slip.

EXAMPLE.

There is a Cube of Stone, whoſe Side is equal to 2 $\frac{1}{2}$ Feet, what's the ſolid Content?

Practice, Set the Side of the Cube 2 $\frac{1}{2}$ Feet accounted on the Slip, to 12 on the girt Line, and againſt 30 the Side of the Cube in Inches (which is equal to $\frac{1}{4}$ of its Girt) ſtands 15 $\frac{1}{2}$ Feet on the Slip, which is the ſolid Content required.

Thirdly, Of the Parallelopipedon.

The Analogy is the ſame as before for the Cube, *viz.*

As the Length of the Parallelopipedon taken in Feet and Inches accounted on the lower Line of the Slip,

Is to 12 accounted in the girt Line,

So is $\frac{1}{4}$ of the Girt of the Parallelopipedon in Inches accounted on the girt Line, to the ſolid Content in Feet, accounted in the Slip.

EXAMPLE.

There is a long Cube or Parallelopipedon, whoſe Length is 17 Feet 9 Inches, and $\frac{1}{4}$ part of the Girt or Circumference 22 Inches $\frac{1}{2}$.

Practice, Set 17 $\frac{3}{4}$ Feet the Length accounted on the Slip, to 12 accounted on the girt Line, and againſt 22 $\frac{1}{2}$ Inches the $\frac{1}{4}$ of the Girt accounted in the Girt Line, ſtands 66 in the Slip, which is the ſolid Content required.

A. But ſuppoſe that the Baſe of the Parallelopipedon is not exactly ſquare, having its Breadth greater or leſſer than its Depth, will by taking $\frac{1}{4}$ of the Girt as before, give the true Solidity.

B. No, you muſt firſt find a mean Proportional between the Breadth and the Depth, and afterwards proceed as when the Breadth and Depth are equal.

A. What do you mean by a mean Proportional?

B. A mean Proportional is a Number, which being ſquar'd or multiplied into its ſelf, produces the ſame Quantity that two given Numbers would do, being multiplied into one another, to which it is a

mean Proportional, or otherwise it is the square Root of the Product produced by the Multiplication of the two unequal Sides into one another.

Suppose the Breadth of the Parallelopipedon be 9 Inches, and Depth 4 Inches: I say, if the 9 be multiplied by 4, the Product will be the 36, and the mean Proportional between 4 and 9, is 6; for 6 times 6 is 36, which is equal to 4 times 9? therefore 6 is the mean Proportional between 4 and 9.

Now for your Supposition, which many others falsly imagine to be true, and thereby very often commit great Mistakes.

Let the Dimensions be as before, *viz.* 9 Inches in Breadth, and 4 Inches in the Depth, and consequently is therefore 26 Inches in Girt. Now if you take $\frac{1}{4}$ part thereof, *viz.* 6 $\frac{1}{2}$ for the Side of the Square, as in the square Parallelopipedon, 'tis plain that it will produce a Content too great, for 6 $\frac{1}{2}$ multiplied by 6 $\frac{1}{2}$ will produce 42 $\frac{1}{4}$ which is 6 $\frac{1}{4}$ too much in the Area, and that being multiplied into the Length would carry on the Error much higher.

Hence 'tis evident, that to measure an unequal Parallelopipedon, there must first be a mean Proportional found, which may be either produced from the square Root of the Area of the Base, or as following.

Set the greater of the two Numbers (as here 9) on the square or girt Line, to the same Number (9) on the Slip, against the less Number (4) accounted on the Slip stands the mean Proportional (6) on the square Line.

Or thus,

Set the less Number (4) on the Slip to the same Number (4) on the square Line, and against the greater Number (9), accounted in the Slip, stands the mean Proportional, (6) on the square Line as before.

These are the most material Rules for finding the Quantity of Masons Works; now I will proceed to the Bricklayer.

Secondly, Upon Bricklayers Work.

A. How are Bricklayers Works measur'd.

B. By the Foot, Yard, Square and Rod.

A. What kinds of their Works are measur'd by the Foot:

B. All manner of rub'd and gaged Work, as Arches over Windows, Quoins, Cornishes, Fascias, &c. all which are measur'd, as directed for Pavements in the Masons Works.

A. And

A. And will the ſame Rule ſerve for the Menſuration of their other Works, which they perform by the Yard or Rod.

B. No, for yard Meaſure, your firſt ſtated Number muſt be 9, and for rod Meaſure 272, which before in foot Meaſure you remember was 12, and therefore it will be neceſſary for you to have a ſmall braſs Stud fix'd in the upper Line of your Stock and Slip at 9, and 272 ¼, whereby thoſe center Points as they are call'd by Workmen, or firſt Numbers, will be readily found.

A. Very well *Sir*, I have prepar'd my Rule with thoſe Studs fix'd at 9, 12, and 272 ¼, now be pleas'd to ſhew their Uſe.

Firſt, Of Yard Meaſure.

Wherein note, that all Works meaſur'd by the Yard, the Dimenſions are taken in Feet and Quarters of Feet.

The Analogy is

As the fix'd Number 9 accounted on the upper Line of the Stock,
Is to the Breadth in Feet accounted on the Slip,
So is the Length accounted on the Stock, to the ſuperficial Content on the Slip.

EXAMPLE 1.

There is a Cellar pav'd with paving Bricks, whoſe Length is 15 Feet ¼, and Breadth 12 Feet ¼, what is the ſuperficial Content thereof.

Practice, Set the Breadth 12 Feet ¼ on the Slip to the fix'd N° 9 on the Stock, and againſt 15 Feet ¼ accounted on the Stock ſtand 21 ⅓ on the Slip, which is the ſuperficial Content requir'd.

EXAMPLE 2.

If 30 Bricks pave one Yard, how many Yards will 630 Bricks pave?

Analogy,

As the Bricks of one Yard accounted in the Stock,
Is to 1 accounted on the Slip,
So is the Number of Bricks given, accounted in the Stock, to the Yards, which they will pave accounted on the Slip.

Practice, Set 30 on the Stock to 1 on the Slip, and againſt 630 on the Stock ſtands 21 on the Slip, which is the number of Yards that 630 Bricks will pave as required.

Pray excuſe this laſt Example, it being a Digreſſion from the former, but ſince 'tis oftentimes uſeful, I thought it not amiſs to inſert.

Secondly,

Secondly, Of square Measure.

Wherein the Dimensions are taken in Feet and quarters of Feet, as before in the yard Measure.

A. What do you mean by a Square?

B. A Square is a square Space, containing 100 square Feet, or it is a Geometrical Square, whose Side is = 10 Feet, and consequently the whole = 100 Feet.

By this Measure all manner of Tyling and Slating is performed, as following.

Analogy,

As the Breadth accounted on the Stock,
Is to 100 accounted on the Slip,
So is the Length accounted on the Slip, to the Content accounted on the Stock.

EXAMPLE

There is a Roof whose Length is 70 Feet, and Depth from the Ridge to the Eves, 15 Feet, what's the Content thereof.

Practice, Set the Breadth 15 accounted on the Stock, to 1 accounted on the Slip, and against the Length 70 on the Slip, stands $10\frac{1}{2}$, the number of Squares therein contain'd, which is = 1050 square Feet.

This Product or Content, is but $\frac{1}{2}$ the Quantity of Tyling, if both Sides of the Roof are equal, therefore the $10\frac{1}{2}$ Squares being doubled, the Content of the whole will be found to be 21 Squares complete.

And here note, that as one Square is	=	100 Feet,
Three Quarters of a Square is	=	75
Half a Square is	=	50
A Quarter of a Square	=	25
Half Quarter of a Square	=	$12\frac{1}{2}$

Thirdly, Of Rod Measure.

B. What is a Rod?

B A Rod is a square Measure, consisting of $272\frac{1}{4}$ square Feet, produced by the squaring of a Rod in Length, *viz.* $16\frac{1}{2}$ Feet multiplied into its self, its Product is $272\frac{1}{4}$; but in Practice the odd $\frac{1}{4}$ is rejected, and 272 Feet only is esteem'd a square Rod.

A. What kinds of Bricklayers Works are measur'd by the square Rod?

B. All

B. All manner of Walls and Chimnies, which tho' of various Thickneſs, yet they are all meaſur'd as ſuperficial Meaſure, being reduc'd to the ſtandard Thickneſs of one Brick and half.

By this Analogy,

As the Length accounted on the Stock,
Is to 272 accounted on the Slip,
So is the Height accounted on the Slip to the Content on the Stock.

Or thus,

As the fix'd Number 272 on the Slip,
Is to the Length accounted on the Stock,
So is the Height accounted on the Slip to the Content on the Stock.

EXAMPLE.

A Brick Wall is 110 Feet in Length, and 9 Feet $\frac{1}{2}$ high at Brick and $\frac{1}{2}$ in Thickneſs.

Practice, Set 272 on the Slip againſt 110 on the Stock, and againſt 9 $\frac{1}{2}$ Feet on the Slip ſtands 383, which is 3 Rod, and 83 Parts of 100, ſuppoſing the ſquare Rod to be divided into 100 equal Parts.

But as theſe Parts thus remaining may be ſomething troubleſome, I therefore recommend the Menſuration of Brick-work to be perform'd by vulgar Arithmetick, in which the Remains will be ſquare Feet.

As for Example,

I will perform the preceeding Queſtion Arithmetically.

Thus,

Multiply the Length of the Wall	110 Feet
By the Height	9 $\frac{1}{2}$
	990
	55
The Product or ſuperficial Content	1045

This Product or ſuperficial Content 1045 muſt be divided by 272, as following,

272) 1045 (3 Rod
816

229 Feet remains.

Now

Now here you see, that the Quantity of Brick-work is 3 Rod and 229 square Feet, which is $\frac{3}{4}$ of a Rod and 25 Feet.

For as 272 Feet is = 1 Rod

$$\text{Therefore}\left\{\begin{matrix}204\\136\\68\end{matrix}\right\}\text{Feet is}\left\{\begin{matrix}=\frac{3}{4}\\=\frac{1}{2}\\=\frac{1}{4}\end{matrix}\right\}\text{Of a Rod}$$

A. But how do you reduce Brickwalls, of one Brick or two Bricks, *&c.* thick to the standard Thickness of Brick and half.

B. By the following Rule, *viz.*

Multiply the superficial Content produced by the Length multiplied into the Height, by the Number of $\frac{1}{2}$ Bricks, which the Wall contains in Thickness, and divide the Product by 3, the Number of $\frac{1}{2}$ Bricks in a Brick and $\frac{1}{2}$ Wall, and the Quotient will be the true Content required.

EXAMPLE 1.

The aforesaid Wall whose Length being multiplied into its Height, and produced 1045 Feet, is of one Brick thick only, what's the Content at Brick and $\frac{1}{2}$ thick reduced.

Practice, The superficial Content	1045
Multiply by 2 the Number of $\frac{1}{2}$ Bricks in Thickness	2
Product at $\frac{1}{2}$ Brick thick	2090

Now divide 2090 by 3, the $\frac{1}{2}$ Bricks in a Brick and $\frac{1}{2}$ Wall, and the Quotient will be 696 Feet $\frac{2}{3}$ at standard Thickness.

3) 2090 (696 Feet at standard Thickness,
18
—
29
27
—
20
18
—
2

Again, Divide 696 by 272, and the Quotient will be 2 Rod 152 Feet which is equal to two Rod and half, and 16 square Feet.

272)

272) 696 (2 Rod,
544

152 Feet

EXAMPLE 2.

The ſame Wall is ſuppoſed to be 2 Brick and ½ thick, what's the Content at ſtandard Thickneſs?

The ſuperficial Content	1045
The ½ Bricks in 2 Brick ½	5
Product in ½ Bricks	5225

3) 5225 (1741 Feet at ſtandard Thickneſs.
3
22
21
12
12
05
3
2

This 1741 being divided by 272, the Quotient will be 6 Rods, 109 Feet.

272) 1741 (6 Rods
1632

109

And ſo in like manner all other Thickneſſes may be reduced to the ſtandard Thickneſs of one Brick and half as required.

Thirdly, Of Carpenters Work.

Carpenters Work, as Roofing, Quartering, Flooring, &c. is perform'd by the Square, and therefore is to be meaſur'd as before taught of the Menſuration of Tyling in the Bricklayers Work.

Fourthly, Of Joyners, Painters, Plasterers, and Paviours Works.

The Plasterer and Paviour perform all their Works by the Yard, as alſo the Joyner and Painter, ſome few Works excepted, which are done by the Foot; and ſince that the manner of foot Meaſure is already declared in the Maſons Works, and the yard Meaſure in the Bricklayers Works, it is therefore needleſs to repeat them again, the Operations being exactly the ſame.

Fifthly, Of Glaziers Work.

B, Glaziers meaſure their Work by the ſquare Foot, and take their Dimenſions in Feet and 100 Parts of Feet, and therefore on the Edge of ſliding Rules, the Foot is generally divided into 100 equal Parts number'd 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, and oftentimes the whole two Foot onwards from 100 to 200.

A. And are the Dimenſions of Glaſs never taken in Inches and quarters of Inches?

B. Very rarely, but when they are

This is the Analogy,

As 144, which is the firſt and fix'd Number for foot Meaſure accounted on the Stock,

Is to the Breadth taken in Inches accounted on the Slip,

So is the Length accounted in Inches on the Stock, to the Content on the Slip required.

EXAMPLE.

A Pane of Glaſs is 31 Inches ½ in Length, and 8 ½ Inches in Breadth, what's the Content?

Practice, Set the Breadth 8 ½ in the Slip to 144 in the Stock, and againſt 31 Inches ½, the Length accounted, in the Stock ſtands 1, 86 in the Slip, which is the Content required.

But ſince this Method is not ſo uſual as the firſt mention'd, which is perform'd by Multiplication only, I ſhall therefore proceed unto it as following. Wherein obſerve,

That as the Foot is divided into 100 equal Parts.

$$\text{Therefore}\left\{\begin{array}{l}\tfrac{3}{4}\text{ of a Foot is}\\ \tfrac{1}{2}\text{ a Foot is}\\ \tfrac{1}{4}\text{ of a Foot is}\\ \tfrac{1}{2}\text{ a quarter is}\end{array}\right\}\text{equal to}\left\{\begin{array}{l}75\\ 50\\ 25\\ 12\ \tfrac{1}{2}\end{array}\right.$$

EXAMPLE

EXAMPLE I.

Suppose a Pane of Glass be 3 Foot, 61 Parts in Length, and 2 Foot 25 Parts in Breadth, what's the Content?

☞ Now always observe to place Feet and Parts in both the Dimensions over each other as following, separating the Feet from the Parts by a Point, and draw a Line underneath them as thus

Feet	P.
3	61
2	25

When you have thus placed your Dimensions, multiply the one by the other, as in common Multiplication, and cutting off four Figures from the Product towards the right Hand, the Remainders to the Left will be the Feet contain'd therein, and those four Figures cut off will be the Parts of a Foot remaining.

See the Operation.

```
    3 61
    2 25
  ------
   18 05
   72 2
  722
  ------
```

Product 8)1225, which is 8 Foot and 1225 Parts of 10000.

A. Pray how comes the Foot to consist of 10000 Parts?

B. By multiplying of 100, the Parts of the Foot in Length, by 100, whose Product is 10000.

Now seeing that one Foot is equal to 10000 Parts,

Therefore	Three quarters of a Foot	is =	7500	Parts
	Half a Foot		5000	
	Quarter of a Foot		2500	
	½ Quarter of a Foot		1250	

By this Table of the Parts of a Foot, the Value of the 4 Figures may be readily discover'd.

Thus the 4 Figures 1225 cut off in the preceeding Example wants 25 Parts of being equal to a ½ Quarter of a Foot, *viz.* 1250 Parts.

Having thus familiarly shewn the Principles of Architecture, as they should be first read by every Lover of sound Work, I shall in the next Part proceed to that so much wanted Part of it (namely Perspective) which teaches how to truly represent the noble Ideas or Designs of the Mind, as if the real Objects themselves were present.

And since that Geometry is the very Basis of Perspective (as well as of all other Arts Mathematical) I must therefore desire the Learner to acquaint himself well therewith, as deliver'd in the first Part hereof, and at his leisure Hours to measure and make Geometrical Plans of Buildings, as also their Geometrical Elevations, which if consider'd as horizontal Plains, instead of vertical or perpendicular Plains, are no more than real Plans, and the more irregular your Choice is, the more delightful your Works will be. By this continu'd Method, the following Part will become easy in all its Parts, and the immense Pleasure that will arise from it, will make amends for all your past Hours.

It is always suppos'd that every Lover of this Art is furnish'd with proper Instruments for that Purpose: But in Case any young Beginners should be destitute thereof, they can be no better furnish'd nor more kindly us'd, than by that truly sound Workman, Mr. *Ben. Scott*, Mathematical Instrument-Maker against *Exeter-Exchange* in the *Strand.*

PER-

PERSPECTIVE

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Or, The Art of Drawing the true Representations of all manner of Lines, Superfices and Solids familiarly demonstrated; Calculated for the Use of Workmen and Others who delight in Designing, Drawing, Painting, Engraving, Architecture, Gardning, &c.

INTRODUCTION.

THE great Complaint made by Men of the Building Trade, for want of a concise and familiar Work of this kind, as also the favourable Reception that my Works on Building have met with, induc'd me to the Performance of this Treatise; wherein I have endeavour'd to render the Art of Perspective in as plain and easy a Manner as it will permit. And I am certain that whoever is desirous of attaining to a perfect Knowledge in this Art, will not fail of Success, provided that they learn each Problem in its proper Order as they proceed, before they begin on the next. For it is as impossible to be perfect in any Art or Science, without first going regularly thro' their Rudiments, as it is to build a strong and massy Structure, without having first laid a good and sure Foundation.

Nor will the young Artist be in the least discourag'd; for in the very Rudiments there is a competent Share of Pleasure, which will induce him to pursue the Study.

Among all the Mathematical Arts I know of, none gives so much Satisfaction as the Art of Perspective; for it is by that, we are not only taught how to truly delineate the Representation of every Object that appears to our Sight, but the Reasons of Sight also is demonstrated, as will appear hereafter in its Place.

There

There are many Treatiſes written on this Art, but none of them has had any Regard to the Workman, who when he's to give in a ſmall Deſign to a Gentleman, (when there's no need for a *Surveyor*) would gladly repreſent the ſame perſpectively, as it will really appear when completed.

It is from the want of this Art being made plain and eaſy to Workmen, that they generally give in their Deſigns Geometrically only; whereby 'tis very difficult for many People to determine, what Effect a Building will have on the Eye of the Beholder when finiſh'd. For whatever Projections or Breaks forward they deſign in their Uprights or Fronts, they are not to be diſtinguiſh'd any otherwiſe than by the Plan, and there's not one Workman or Gentleman in twenty can determine from a bare Plan the real Effect of a Geometrical Elevation.

And ſince that a Perſon cannot be a complete Architect, without being Maſter of this Art, I will therefore proceed in the moſt eaſy manner as follows.

SECTION I.

Of the principal Lines and Points incident to the Art of Perſpective.

THE Lines under which all Repreſentations are terminated, are principally five, *viz.* The Baſis or Ground Line G E, *Fig.* 1. Plate 2. The viſual or horizontal Line F H, the diagonal Lines F E and G H, the radial Lines I G and I E, and the traverſe Line K L.

Firſt, The baſe Line G E, is the firſt Line that is given in all Operations, and repreſents the Surface or Ground on which we ſtand to view an Object, and is therefore call'd the Baſe or ground Line, and is always fixt.

Secondly, The viſual or horizontal Line F H, *Fig.* 1. is undeterminable as to its Height, according to the Situation of the Eye of the Beholder; for whatever Height the Eye of the Beholder is above the Baſe or ground Line, ſo high muſt this viſual or horizontal Line be plac'd.

As for Example, *Fig.* 2.

1. Let D Z repreſent the Baſe or ground Line, and let the Line *a b*, be the Height of a Boy's Eye above the ſame, then if the right Line L K be drawn parallel to D Z, thro' the Point *a*, it will be the horizontal

Line to the Boy *a b*, whilst his Eye remains in that Position above the ground Line.

2. But if the Boy's Height should be increas'd to the Height of a Man, &c. as the Height of the Line *d c*, having his Eye at *d*, then will the horizontal Line be rais'd as high as G I, *viz.* equal to the perpendicular Height of the Eye *d c*.

3. *Again*, When we ascend any Eminence, as the Pedestal L D, and stand thereon, as the Line E, the horizontal Line will be also rais'd as the Line B A.

Hence 'tis evident that the horizontal or visual Line, is a Line parallel to the Basis or ground Line, at the Distance of the Eye's perpendicular Height above the same, be that more or less.

Thirdly, The diagonal Lines F E and H G, drawn from the Points of Distance F H, are also undeterminable; because those two Points may be taken at Pleasure in any Parts of the horizontal Line, as will appear when I come to their Definition. These Diagonals are always *Fig.* 1. drawn from the two extream Points of the Object that we are to represent, as from G E, to the two Points of Distance aforesaid.

Fourthly, The radial Lines are I G and I E; drawn from I the Point of Sight, to the extream Points (as before was said of the Diagonals) of the Object G E. These Lines are call'd radial Lines, as proceeding from the Point of Sight I (as two Rays of Sight) unto the Extreams of the Object G E.

Fifthly, The right Line K L being drawn from the Points K and L, where the radial Lines intersect the Diagonals, is the traverse Line of the Square G E Z X, and therefore as the traverse Line K L represents the Side of the Square Z X, as it appears to the Eye at F or H; so likewise will the Line K G represent the Side G Z; the Line L E the Side E X; and consequently the Line G E common to both; and thus the Geometrical Square G E Z X, is reduc'd to the Perspective Square K L G E. Wherein observe,

First, That the Line G E is common as well to the Perspective Square K L G E, (being one Side thereof) as to the Geometrical Square G E Z X.

Hence it follows, that if a Perspective Square be given, as K L G E to be transform'd or reduc'd into a Geometrical Square, there's no more requir'd in the Operation, than to describe a Geometrical Square, whose Sides respectively are equal to the ground Line of the Perspective Square G E.

And

And since that the right Lines G X and Z E, are the Diagonals of the Geometrical Square G E X Z, and are represented in the Perspective Square by the Diagonal Lines K E and L G, therefore it is, that I call the Lines G H and E F Diagonals, (as before,) for they are no other than the Diagonals G L and K E, continu'd on until they meet the horizon or visual Line F H.

N.B. It is to be further noted, that all Lines drawn from the Point of Sight I, to the ground Line G E, are radial Lines, as the Lines I G and I E are, because they proceed from the Point of Sight I in the same manner.

But that you may not be at a Stand to know how to distinguish the one from the other, I call the two radial Lines I G and I E, *Principal Radials*; the Line I *d*, which proceeds from the Point of Sight directly thro' the Center of the Squares *a*, direct radial, and all others, as I *n*, I *a*, *&c.* I call *Secondary Radials.*

It is also to be noted, that all Lines which are drawn parallel to the ground Line, as *o r*, *&c.* are *traverse* Lines, as the traverse Line K L, but as K L bounds or terminates the Perspective Square, therefore it is to be call'd the *principal Traverse*, and all others *Diametrical Traverses.* The Learner must likewise observe, that the Angles K L G E are to be consider'd as all equal to one another, as the Angles of the Geometrical Square G E Z X are, which they represent, that is, tho' the Angles K and L, which are Geometrically speaking, both obtuse Angles, and the Angles G and E both acute Angles, yet both the Angles K L and G E of the Perspective Square K L G E, are to be respectively consider'd as right Angles, each containing 90 Degrees, as the Angles of the Geometrical Square G E Z X, which the Angles K L G E do represent.

Fig. 1.

The same is to be understood of all the radial Lines, which are drawn from the Point of Sight upon the ground Line G E.

Again, Tho' the radial Lines are all drawn from the Point of Sight I, and therefore grow farther from one another, the more remote they are continu'd from the Point of Sight, and consequently by their Inclination form divers Angles, yet when those radial Lines enter the Perspective Square at the principal Traverse, they are then to be all conceiv'd as Lines parallel to one another, as the parallel Lines *n t*, *d w*, and *a q*, which they represent, cutting as well the principal Traverse, diametrical Traverses and ground Line, all at right Angles, as the parallel Lines *n t*, *d w*, and *a q*, do the right Lines G E, *g g*; and Z X of the Geometrical Square G E Z X.

These

These Definitions being perfectly underſtood, the Learner will find the greateſt and moſt difficult Part of his Task over, and all that is following very pleaſant.

Having done with the Lines, I ſhall now proceed to the Definition of the Points, which I have already ſpoke of, in the preceding Paragraphs.

The neceſſary Points us'd in Perſpective are principally three, *viz.* the Point of Sight I, and the Points of Diſtance F and H. Plate 5. Fig. 1.

1. The *Point of Sight* is ſometimes call'd the Point of View, but not with Regard to its being, (as falſly underſtood by ſome) the Point of our Eye, (or *Retina*) from whence our Sight proceeds, or where all Objects are painted which we behold; but the contrary, 'tis the extreameſt Point that our Sight can reach (in the Horizon) to diſtinguiſh plainly the Form or Shape that Objects appear to be of. Hence it happens, that the Point of Sight is always proportionable to the Strength, Goodneſs, and Form of the Eye; and therefore 'tis neceſſary in many Works that are to be ſeen at a great Diſtance to conſult the Strength of that Eye which is to view the Objects, becauſe ſome People (as before-ſaid) can ſee farther and ſtronger than others. But however, that is more the Concern of Hiſtory Painters than the direct Buſineſs of an Architect, to whom I am now ſpeaking.

This Point of Sight is not fix'd, but may be taken or plac'd at Pleaſure in any part of the horizontal Line, wherein 'tis always plac'd.

When an Object is to be view'd directly in the Front, as the Geometrical Square G E Z X, *Fig.* 1. or directly oppoſite to an Angle, as the Geometrical Square *q s t x*; *Fig.* 5. the Point of Sight muſt be plac'd exactly againſt their Middles or centeral Lines, as at I and *l*. But when an Object is to be ſeen or view'd on one Side, as the Geometrical Squares *i k l m*, and *p q r s*. *Fig.* 6. then the Point of Sight may be plac'd on one Side thereof, as at *b* or *e*, and the Lines *b i*, and *b k* ſhall be the principal Radials, as *b f*, *b g* are in the direct View, *Fig.* 7. and the like is to be underſtood of the Radials *e p* and *e q* of the Square *p q r s*.

Secondly, The Points of Diſtance *a c* and *d f*, are alſo undeterminable, and are therefore plac'd according to the Diſtance that the Beholder is from the Object, but are always at equal Diſtances on each Side of the Point of Sight, be their Diſtances from thence as they will.

The nearer we stand to an Object whose Surface or upper Part lies under the Horizon, the more we see of it, and contrarily, the more remote we are, the less we see of it, for when we are near, the Radials looks more direct upon the Surface, and makes larger Angles there-with, than when view'd at a greater Distance. This is plainly demonstrated in *Fig. 6. Plate 5.* where the Geometrical Square *k i m l*, being view'd at *c*, the Radials are intersected at *h* and *g*, and the Square *k i m l*, is represented by the Square *h g k i*. But if the Eye is mov'd back to 4, then the Square *k i m l* is represented by the Square 5 6 *k i*, which is much lesser than the Square *h g k i*. For as the Traverse *h g* is drawn parallel to the Base or ground Line from the Point *h*, where the Diagonal *c i* cut the Radial *b k*, so likewise is the new traverse Line 5 6 drawn parallel to the ground Line from the Point 5, where the new diagonal Line 4 *i* cuts the Radial *b k*, whereby the appearance of the Part *g h* 5 6 does then in Effect disappear, but the remains 5 6 *k i* hath still the same Equality in appearance to the Geometrical Square *k i m l*, being view'd at 4, as the whole *h g k i* had, when seen at *c*.

This Diminution is caus'd (as I said before) by the near Inclination of the Rays of Sight to the Surface of the Object. For the Line 4 *i* having a greater Inclination to the Surface or ground Line *k i* than the Line *c i* hath, does form a lesser Angle, which as aforesaid, intersects the radial Line *b k* in 5, and then remains the appearance of the Square 5 6 *k i* as exhibited.

Now seeing that the more or less appearance of an Object wholly depends upon the Distance of its being view'd; the Learner must maturely consider what Quantity he would have represented, and place his Points of Distance accordingly.

Note, That besides these three preceeding Points, there are many others, which occur in some particular Operations, which are call'd *accidental Points*. But as these are not necessary in all Designs, I shall omit speaking any more of them, until I come to the several Operations wherein they are us'd.

Having thus familiarly laid down the Names, Natures and Uses of the principal Lines and Points, by which all superficial Figures and solid Bodies are represented; I shall in the next Place proceed to the several Uses here requir'd.

SECTION

SECTION II.

Of lineal Perſpective.

PROBLEM I.

THE Geometrical Square *g f i h*, whoſe Sides are each = 4 Feet, the Height of the Eye or horizon Line *c a* five Feet, and Points of Diſtance *c b* and *b a*, each = 50 Feet being given, to find the Perſpective Appearance of the Square *g f i h*, in a direct view at *b*. Plate 5. Fig. 7.

PRACTICE.

1. Draw the ground Line *l k*, and about the middle of it deſcribe the Geometrical Square *f g h i*, making the Sides each equal to 4 Feet (by any Scale of equal Parts.)

2. Draw *a c* parallel to *l k*, at the given Height of the Eye above the Object, *viz.* 5 Feet, which will be the horizon Line to your Object *f g h i*.

3. Biſect the Side of the given Square *f g* in *m*, and on *m* raiſe the perpendicular *m b*, cutting the Horizon *a c* in *b*, ſo will the Point *b* be the Point of Sight requir'd in the Horizon.

4. Make *b a* and *b c* each equal to 50 Feet the given Diſtance, then will the Points *a* and *c* be the two Points of Diſtance requir'd.

5. Draw the two Radials *b f* and *b g*, and the two Diagonals *a g* and *c f*, which will interſect each other in the Points *d e*, and then drawing the principal Traverſe Line *d e*, you will complete the Perſpective Appearance (or Square *d e f g*) of the Geometrical Square *f g h i*, as requir'd.

PROBLEM II. *Fig.* 8.

The ſame Geometrical Square as in the preceeding Problem being given with the Points of Diſtance the ſame, to find the Perſpective Appearance in a direct View when the horizon Line *a b* is elevated above the ground Line *g h*, 3 Feet inſtead of 5 Feet, as before.

PRACTICE.

1. Set off by a Scale of equal Parts all the Requiſites belonging hereunto, as before in the preceeding Problem, only inſtead of drawing the horizon Line at the parallel Diſtance of 5 Feet, here you muſt

draw it at the parallel Distance of 3 Feet, and then placing the Point of Sight *z* directly against the Square *g h i k*, with the Points of Distance *c d*, and drawing the Radials *z g*, *z h*, the Diagonals *d h*, *c g*, and the traverse Line *e f* you will complete the Perspective Appearance, (or Square) *e f g h* of the Geometrical Square *g h i k*, being view'd at the Distance of *z d* with the Horizon three Feet above the ground Line, as requir'd.

Fig. 9, Is the same Geometrical Square, view'd at the same Distance as in the two proceeding Examples, having its horizon Line *a d* but one Foot above the Base or ground Line *g k*; from whence it appears that the nearer the horizon Line (or Eye) is to the ground Line, the less Quantity of the Object appears, as before was said with respect to Distance.

Now seeing the Appearance of Objects do thus diminish by the nearer and nearer approach of the horizon Line to the ground Line, it therefore follows that when the horizon Line is so much depress'd as to be in the very Plain of the ground Line, the superficial Object or Geometrical Square *e f g k* will then have the Appearance but of a right Line only, without the Dimensions of its Sides, under which it appear'd before.

PROBLEM III. *Fig.* 10. *Plate* 6.

A Geometrical Square whose Situation is above the Horizon, with the Horizon and Points of Distance given, to find its persective Appearance in a direct View.

PRACTICE.

1. Let the Geometrical Square *a b c d* be given, and let it be plac'd 5 Feet above the horizon Line *x z*, which is also 5 Feet above the ground Line *m y*, and let the Points of Distance be as before, *viz.* each 50 Feet from the Point of Sight *h*.

2. Draw *m y* for the ground Line, and at the parallel Distance of 5 Feet (which is the suppos'd Height of the Eye above the same) draw the Horizon *x z*.

In any Part of it as at *h*, raise the Perpendicular *h o*, which make equal to 5 Feet, the given Height of the Geometrical Square *a b d c*, above the Horizon *g z*, and at the Point *o*, delineate the Geometrical Square *a b d c*, making each Side of it equal to 4 Feet as in the preceding Examples.

This done, the Point *h* will be the Point of Sight, being the direct Point of View in the Horizon before the Geometrical Square *a b c d*, and


Horizon
Fig. III
Horizon
Horizon
Fig IIII
Ground line
Point of Distance
Point of Sight
Point of Distance
Direct
at an
View
Angle
Radial
Radial
Diagonal
Diagonal
Diagonal
Diagonal
Traverse
Fig V
Traverse
Basis &c.
Ground line
Fig. I.
Horizon
Horizon
Radial
Radial
Diagonal
Diagonal
Traverse
Ground Line
Fig VI
Fig. VI
Fig VIII
Horizon
Fig VII
3 Feet
5 Feet
Horizon
Ground
line
Fig. IX

and then setting off 50 Feet on the Horizon from *h* to *i* and from *h* to *g*, you will have the Points of Distance given.

3. From the Point of Sight *h* to the hithermost Side of the Square *d c*, (which must now be call'd the aireal Line (as being above the Eye) instead of the ground Line, on which it in the other Examples stood) draw the Radials *c h* and *d h*, also from the Points of Distance *g* and *i*, draw the Diagonals *d g* and *c i*, which will intersect the Radials in the Points *e* and *f*. Then drawing the right Line *e f*, it will complete the Perspective Appearance *d c f e* of the Geometrical Square *a b d c*, as requir'd.

☞ The Learner must here observe, that when an Object is plac'd no higher above the Horizon than another equal to it, is below the Horizon, that then their Perspective Appearances are equal; for the Geometrical Square B P G D, which is plac'd at the same Distance below the Horizon *g z* and equal to *a b c d*, hath its Perspective Appearance H I B P equal to the Perspective Appearance *c d e f* of the Geometrical Square *a b d c*. For since that the Radials *h c* and *h d* are equal to the Radials *h* B and *h* P, and the Point of Sight *h* common to them all, and the opposite Angles at *h* equal; and since that the Diagonals *g d* and *c i* are respectively equal to *g* P and *i* B, as also equal to one another, and their alternate Angles being equal likewise, therefore the Perspective Squares (or Appearances) *a b d c*, and H I B P, are equal to one another, *which was to be demonstrated.*

Seeing there's no Difference in representing the Appearance of Objects plac'd above the Horizon from those plac'd below it, the Name of the ground Line excepted, which is in such Cases chang'd to the Term of *Areal*, as being above the Eye, (but is yet a ground Line to the Object) I shall now proceed to the manner of representing the Perspective Appearance of the same Geometrical Square being view'd on the Side; that is to say in an oblique Position, as *Fig.* 11.

PROBLEM IV.

A Geometrical Square being given with the Height of the Horizon, Points of Sight and Distance in an oblique Position, to find its Perspective Appearance.

PRACTICE.

1. Let *a b c d* be the given Square, also let *f g* be the horizon Line at the parallel Distance of 5 Feet from the ground Line *m a*, and let *e* be the given Point of Sight remov'd 40 Feet from the direct Point *m*, before us'd towards the right Hand. *Fig.* 11.

2. Set

2. Set off on each Side of the Point of Sight *e*, the two Points of Distance *f e*, and *e g* at the given Distance of 50 Feet each, then drawing the Radials *e a* and *e b*, as also the Diagonals *f a* and *g b*, they will intersect one another in the Points *m* and *n*; andthe traverse Line *m n* being drawn, will complete the Perspective Square *m n a b*, which is the Appearance of the Geometrical Square *a b c d*, as it appears when view'd in the oblique Position at *e*, as requir'd.

Fig. 12 and 13 are Representations of the same Geometrical Square, as it appears when view'd at the same Distance as the preceeding, but with two different Horizons; that of *Fig.* 12 having the Horizon fix'd at 3 Feet above the ground Line, and that of *Fig.* 13 at one Foot above the ground Line.

Hence you see that in oblique Views the Appearance of the Object is diminish'd as the Horizon approaches the ground Line, in the very same manner as was before said of direct Views.

Now, what is here deliver'd with Respect to the Object having its Point of Sight fix'd towards the right Hand, the same is to be also understood, when 'tis requir'd to be plac'd on the left Hand of the direct View, as exhibited by *Figures* A B and C, *Fig.* 14, 15 and 16, where the preceeding Appearances are represented by the very same Lines and with the same Horizons, having their Points of Sight *a b* and *c* on the left Hand of the direct Views *d e f*.

Again, The same is to be likewise understood of the Geometrical Square *o n k l*, *Fig.* 17, when view'd above the Horizon, as will appear by the following Problem.

PROBLEM V.

A Geometrical Square being given, and plac'd above the Horizon (or Eye) with the Points of Sight and Distance, to find its Perspective Appearance in an oblique or side View.

PRACTICE.

Let *o n k l* be the given Square, plac'd five Feet above the Horizon *d a*, also *c* be the given Point of Sight, plac'd 40 Feet from the direct Point of View *p*, towards the left Hand, and *e* and *d*, the two given Points of Distance, each 50 Feet from the Point of Sight *c*. This done, draw the Radials *k c* and *l c*, also the Diagonals *k e* and *l c*, which will intersect each other in *m i*; then drawing the traverse Line *m i*, you'll complete the Perspective Appearance *k l m i* of the Geometrical Square *n o k l* as requir'd.

From

Hence 'tis plain, that this Operation where the Object is plac'd above the Horizon (or Eye) is in all respects, exactly the same with that of Problem 4, *Fig.* 11. And the Reason why I plac'd it here again in this manner, was for the Learner's Information, and to prevent his being at a Stand at such Times when such Operations are requir'd to be done.

Having thus shewn the various Methods of representing the Geometrical Square, I will now proceed to shew the Manner of representing parallel Lines therein, and then proceed to other superficial Figures.

PROBLEM VI.

A Geometrical Square with divers parallel Lines perpendicular to the Horizon, being given, with the Points of Sight and Distance, to find their Perspective Appearance in a direct and oblique View.

First, For the direct View. Plate 6. Fig 18.

PRACTICE.

1. Let the Geometrical Square *a b c d*, with the Parallels *a f*, *b f*, *c f*, and *o f*, be the given Square; the Line *i k* the given Horizon; the Point *g*, the given Point of Sight, and the Points *h k* the given Points of Distance.

2. Draw the Radials *a g* and *g b*; also the Diagonals *k a* and *h b*, and complete the Perspective Square *f e b a*.

3. From the Points *a b c o*, where the parallel Lines terminate at the ground Line *a b*, draw the right Lines or *secondary Radials a e*, *b e*, *c e*, and *o e*, terminating them in the traverse Line *e f*, which will complete the direct View of those parallel Lines as required.

Secondly, For the oblique View.

1. Let the Geometrical Square *f g h i* be the given Square, Fig. 19. and the parallel Lines *k k*, *l l*, *n n*, and *m m*, the given Parallels. Also let *b* be the given Point of Sight, and *a c* the two given Points of Distance.

2. Lay a Ruler from the Point of Sight *b*, to the Points *k l n m*, in the ground Line *g f*, and draw the right Lines, or secondary Radials *k* 1, *l* 2, *n* 3, *m* 4, which are the Perspective Appearances of the parallel Lines *k k*, *l l*, *n n*, and *m m*, being view'd in an oblique Position at *b* as requir'd.

PROBLEM VII.

A Geometrical Square with divers parallel Lines, parallel to the ground Line being given, with the Points of Sight and Distance to find their Perspective Appearances in direct and oblique Views.

First,

First, For the direct View.

PRACTICE.

1. Let the Geometrical Square *a b c d*, with the parallel Lines 1 1;
Fig. 20. 2, 3; 3, 4; 4, 5; be given, and let *f* be the given Point of Sight, and the Points *e* and *g*, the given Points of Distance.

2. Draw the Radials *f a*, and *f b*; also the Diagonals *e b* and *g a*, and complete the Perspective Square *i h b a*.

3. Draw either of the diagonal Lines of the Geometrical Square *a b c d*, as *a d*, and from the Points *k*, *m*, *q*, *r*, where the several Parallels intersect the Diagonal *a d*, draw up the Parallels *k l*, *m n*, *q o*, *r p*, unto the ground Line *a b*; then laying a Ruler from the Point of Sight *f*, to the several Points *p*, *o*, *n*, *l*, it will cut the Diagonal *h b*, in the Points *s t u w*; and if through these last produc'd Points, you draw right Lines parallel to the ground Line *a b*, they will be the Perspective Appearance of the parallel right Lines of the Square *a b c d*, being seen in a direct View from the Point of Sight *f*, as requir'd.

Secondly, For the oblique View.

PRACTICE.

Plate 7. Fig. 21. 1. Let the Geometrical Square *f g h i*, with the Parallels 7, 1; 7, 2; 7, 3; 7, 4; be given, and let *b* be the Point of Sight, and the Points *a* and *c*, the Points of Distance given.

2. Draw the Radials *b f*, and *b g*, also the Diagonals *a g*, and *c f*, and complete the Perspective Square *d e f g*.

3. Draw the Diagonal *g h* of the Geometrical Square, which will intersect the given Parallels in the Points *k l m u*; from which draw up (as before) the pricked perpendicular Lines *k o*, *l p*, *m q*, and *u r*, terminating at the ground Line *g f*, in the Points *o p q r*.

4. From the aforesaid Points *o p q r*, draw the secondary Radials *o s*, *p t*, *q u*, *r w*, untill they meet the diagonal Line *f c* in the Points *s t u w*, thro' which draw the Diametricals *s x*, *t x*, *u x*, and *w x*, parallel to the ground Line *g f*, and they shall be the Perspective Appearance of the parallel Lines 7, 1; 7, 2; 7 3; and 7, 4; given in the Geometrical Square *f g h i*, as requir'd.

PROBLEM VIII.

A Geometrical Square with divers parallel right Lines therein, cutting one of the Diagonals at right Angles, being given to find their Perspective Appearances in a direct and oblique View.

First,


Fig. XIIII.
C
Fig XV.
B
Fig XVI.
A
Aireal line
5 Feet
Horizon 50
50 line
5 Feet
Fig X.
Point of Distance
Point of Sight
Point of Distance
Diagonal
Radial
Radial
Diagonal
Traverse
3 Feet
5
Fig XIII.
Fig XII.
Fig XI.
Ground or aireal line
40
Fig XX
Fig XIX
Fig XVIII
Fig XVII

First, For the direct View.

PRACTICE.

1. Let the Geometrical Square *h m i h*, with the parallel right Lines *a* 1, *b* 2, *c* 3, *e* 4, *d* 5, (which cut the Diagonal *h l* at right Angles, at the Points 1 2 3 4 5.) be given, and let *o* be the given Point of Sight, and the Points *p* and *q*, the given Points of Distance. Fig. 22.

2. Draw the Radials *o h*, and *o i*, also the Diagonals *p i*, and *q h*, and complete the Perspective Square *h m i h*.

3. From the Points 1 2 3 4 5 in the Diagonal *h l*, draw up the prick'd perpendicular Lines 1 *a*, 2 *b*, 3 *c*, 4 *d*, 5 *e*, unto the ground Line *h i*.

4. Lay a Ruler from the Point of Sight *o*, unto the Points *a b c e d* and draw up the prick'd secondary Radials, until they meet the Diagonal *m i* (which represents the Diagonal *h l*) in the Points *x x x*, &c.

Lastly, From these last produc'd Points *x x x*, &c. to the Point of Distance *q*, lay a Ruler, and draw the right Lines *z a*, *z a*, &c. which are the Perspective Appearances of the given Parallel Lines 1 *a*, 2 *b*, 3 *c*, &c. as required.

Secondly, For the oblique View.

PRACTICE.

1. Let the Geometrical Square 1 2, *p*, *q*, with its Diagonal 1 *g*, divided (as before) by the parallel right Lines 6 *b*, 6 *a*, *p* 2, *n l* and 6 *m h*. Fig. 23.

2. From the Points *i h k l m*, where the Parallels cut the Diagonal 1 *g*, draw up the prick'd Perpendiculars *i a*, *h b*, *h c*, *l d*, *m e*, until they meet the ground Line 1, 2.

3. Lay a Ruler from the Point of Sight 6, to the Points *a b c d e*, and draw up the secondary Radials until they meet the Diagonal 4, 2; (which represents the Diagonal 1 *g*.) in the Points *s t u w x*.

4. Lay a Ruler from the last produc'd Points *s t u w x*, unto the Point of Distance 7, and draw the right Lines 7 *s*, 7 *t*, 7 *u*, 7 *w*, and 7 *x*, which are the Perspective Appearances of the Parallels 6 *b*, 6 *d*, *p* 2, *n l*, *m h*, in the given Geometrical Square 1 2, *g p*, as required.

PROBLEM IX.

A Geometrical Square with a right Line drawn in it, terminating on each Side in an oblique Position, being neither parallel to any of its Sides, Diagonals, or Diameters, being given, to find its Perspective Appearance in a direct and oblique View. Fig. 24.

N *First*,

First, For the direct View.

1. Let the Geometrical Square *f g k l*, with the oblique right Line *h i* be given, and let *b* be the given Point of Sight, and *a c* the given Points of Distance.

2. Draw either of the Diagonals of the Geometrical Square, as *g k*, and from the Point *h*, where one End of the given Line terminates, draw the right Line *q h*, parallel to the ground Line *f g*; and from the Point *q*, where *q h* cuts the Diagonal, draw up the right Line *q r* parallel to *g h*, which terminate in the ground Line *f g* in the Point *r*. This done, lay a Ruler from the Point of Sight *b*, to the last produc'd Point *r*, and it will intersect the Diagonal *f e* in *s*. Then from the Point *s*, draw the right Line *s t* parallel to the traverse Line *d e*, cuting the Side of the Perspective Square *e g* in the Point *t*, which Point *t* represents the Point *h* in the Side *g l* of the Geometrical Square *f g k l*, and is one End or Termination of the given Line *i h*. The other Point or End of the Line *i*, represented by *u*, is found as following.

Where the given Line *h i* intersects the Diagonal *g k* in *m*, draw the right Line *m o* up to the ground Line *f g*, and parallel to *f k*. Then laying a Ruler from *b* the Point of Sight, to *o* the last produc'd Point in the ground Line, and it will cut *f e*, the Diagonal of the Perspective Square, (which represents the Diagonal *k g* of the Geometrical Square) in the Point *p*. And if from the Point *t* thro' the Point *p*, you draw the right Line *t p u*, it will be the Appearance of the given Line *h i*, being view'd in a direct View as requir'd.

Fig. 25. *Secondly, For the oblique View.*

1. Let the Geometrical Square *c d e f*, with the oblique Line *l k* be given, and let *h* be the given Point of Sight, and *g i* the given Points of Distance.

2. Draw the parallel prick'd Lines *n l*, *n q* and *m o*, as before directed in the direct View.

3. Lay a Ruler from *q* to the Point of Sight *h*, and it will cut the Diagonal *c b* in *r*; and then drawing *r s*, parallel to the traverse *a b*, the Point *s* will represent the Point *l* in the Geometrical Square *c d e f*, which is one End of the given Line *k l*.

4. A Ruler laid from the Point of Sight *h* to the Point *o*, will cut the Diagonal *c b* in the Point *t*, which represents the Point *m* in the Diagonal *e d*, of the Geometrical Square *c d e f*.

Lastly, A right Line being drawn from the Point *s* thro' the Point *t*, until it meet the Side *a c* in *p*, will be the Perspective Appearance of

of the given oblique situated Line $l\ k$ being view'd in an oblique View as requir'd. *N. B.* These preceding Problems are plac'd here in the beginning, in order to let the Learner see how easy the Performance of such Operations are. But as all these, and most of the following are confin'd within a Geometrical Square, which in Practice doth not always happen; and since that all superficial Figures are bounded or terminated by Points, I shall before I proceed to the following Section, give one general Rule, *How to represent any Point or Points given in any regular or irregular Figure or Figures in any View as required.*

Let the Point r of the Geometrical Square $q\ p\ s\ r$. (*Fig.* 6 Plate 5.) be a single Point given at Random, to find its Perspective Appearance in an oblique View, the Points of Sight and Distance being $f\ e\ d$.

RULE.

Always from the given Point (as r) let fall a Perpendicular (as $r\ p$) to the ground Line, and making the perpendicular Radius on the Point where it meets the ground Line describe a Semi-circle, (as $q\ r$ 2) then drawing right Lines from the two Points of Distance ($f\ d$) to the Extreams of the Semi-circle on the ground Line (q and 2,) they will intersect each other in (n,) the given Point requir'd. And so in like manner may the Perspective Appearances of many Points be represented most easily, let their Situations or Positions be as they will.

☞ I must here advise the Learner to be very perfect in all the preceding Problems before he proceed any further, for on them depends all the Operations contain'd in the following Problems.

SECTION III.

Of superficial Perspective:

Or the manner of representing all kind of plain Geometrical Figures, as Triangles, Circles, Parallelograms, Polygons, &c.

PROBLEM I.

THERE's an equilateral Triangle $k\ l\ o$ given, and 'tis requir'd to find the Perspective Appearance thereof, when seen in a direct and oblique View, the Horizon being 5 Feet above the ground Line, and the Points of Distance each 50 Feet from the Point of Sight.

 First,

Fig. 26. *First, For the direct View.*

PRACTICE.

1. Let the equilateral Triangle *l k o* be given, as also the Point of Sight *c* at the given Height above the ground Line *k l*, and the Points of Distance *h* and *g*, each 50 Feet from *c*.

2. Describe the Geometrical Square *l k p m*, making the Side of the equilateral Triangle *g l o* one Side; and draw the Diagonal *k m*.

3. From the Point *n*, where the Diagonal *k m* cuts the Side of the Triangle *l o*, draw up the right Line *n d* parallel to *l m*. This done, draw the Radials *c l*, *c k*, and the Diagonals *h k* and *g l*, and complete the Perspective Square *e f l k*.

4. Lay a Ruler from *c* the Point of Sight, to *d*, and it will cut the Diagonal *l g* in *a*, then will this last produc'd Point *a* represent the Point *n* in the Diagonal *k m*. And since that the Side of the given Triangle *l o* passes thro' *n*, and terminates at the Radial *c o* in *o*: Therefore from the Angle *e*, in the Perspective Square *e f l k*, thro' the Point *a*, draw the right Line *e a i*, terminating in the Radial *c o* in *i*.

Lastly, Since the other Side of the Triangle *k o* terminates in *k* and *d*, which Points are represented in the Perspective Square by the Points *f* and *i*, therefore draw the right Line *f i*, and the Triangle *e f i* will be the Perspective Appearance of the equilateral Triangle *g l o* being seen in a direct View as requir'd.

Secondly, For the oblique or side View.

PRACTICE.

1. Let *g h p* be the given Triangle, and let the Geometrical Square *g h, s r* be made as before with the Diagonal *h s*, drawn thro' the Side *g p*, cutting it in *o*; also let the prick'd Line *o k* be drawn up to *k*, parallel to *g s*, and draw the Radials *c g*, *c h*, and Diagonals *a h*, *d g*, and complete the Perspective Square *e f g h*.

Fig. 27. 2. Lay a Ruler from the Point of Sight *c* to *k*, and draw up the prick'd Radial *k i*, until it meet the Diagonal *g f* in *i*, (as before in the direct View.) Then drawing the right Line *e n*, thro' the Point *i*, until it meet the direct Radial *x m* in *n*, that Line shall represent the Side *g p*, which passes thro' the Diagonal *s h* in *o*, in the same manner.

Lastly, Draw the right Line *f n*, and 'twill complete the Representation or Perspective Appearance *e f n* of the equilateral Triangle *g h p*, being seen in an oblique View as requir'd.

PRO-

PROBLEM II.

There's an Isoceles Triangle *a b c* given, and 'tis requir'd to find the Perspective Appearance in a direct and oblique View, having the Points of Sight and Distance given as before.

PRACTICE.

First, For the direct View.

1. Let the Isoceles Triangles *a b c* be given.

2. Upon the Base thereof *a c* complete the Geometrical Square *a c g x*, with its Diagonal *c x*, and let *a c* be the ground Line given.

Also let *s* be the given Point of Sight, and *w t* the Points of Distance, and draw the Radials *s a* and *s c*, also the Diagonals *w c* and *t a*, and complete the Perspective Square *f m a c*. Fig. 28.

3. From the angular Point *b*, of the Isocles Triangle *a b c*, draw the prick'd Line *f f* parallel to the ground Line *a c*, until it cut the Diagonal in *p*; and then, from *p* draw the right Line *p i* up to the ground Line *a c*, and parallel to *c g*.

4. Lay a Ruler from *s* to *i*, and draw up the Radial *i o*, until it cut the Diagonal *a t* in *o*, and from *o* draw the prick'd Line *q q* parallel to the Traverse *f m* until it intersect the centeral Radial *s d* in *n*.

Lastly, Since the Point *n* in the Perspective Square represents the Point *b*, the angular Point in the Geometrical Square, therefore draw the two right Lines *f n*, *n m*, and they will represent the Lines *a b* and *b c*, and complete the Perspective Appearance of *a b c*, being seen in a direct View as requir'd.

Secondly, For the oblique View.

1. Let *a b f* the given Triangle, and *a f d z*, the Geometrical Square describ'd upon the Base *a f* of the Isoceles Triangle, with the Diagonal *d f* drawn as before, and let *k* be the determin'd Point of Sight, and the Points *i h*, the Points of Distance. Fig. 29.

2. Draw the Radials *k a* and *k f*, also the Diagonals *i f* and *h a*, and complete the Perspective Square *f g f a*.

3. Draw the parallel Line *g g* from the angular Point *b*, until it cut the Diagonal *f d* in *e*, (as in the direct View;) also draw up the right Line *e i* to the ground Line *f a*, and draw the Radial *k o*, until it intersect the Diagonal *a h* in *o*.

From *o*, draw the right Line *g p* parallel to the traverse Line *f g* until it cut the direct Radial *m i* in *n*, then will this Point *n* represent the angular Point *b* in the Geometrical Square *a f d z*; and if you draw the right Lines *g n* and *n f*, the Triangle *f n g* will represent the Isoceles Triangle *a b f* given as requir'd

Note,

Note, That if the Angle *b* of the Isoceles Triangle given, had terminated at *x*, its Perspective Appearance had been limited in the direct View by the Triangle *f z g*.

PROBLEM III.

There is scalenum Triangle *a b c* given, and 'tis requir'd to find its Perspective Appearance, being seen in a direct and oblique View, having the Points of Sight and Distance given.

First, For the direct View.

PRACTICE.

1. Let the scalenum Triangle *a b c* be given, and let its Base *a c* represent the ground Line, also let the Point *e* be the given Point of Sight, and the Points *d* and *f* the given Points of Distance. *Fig.* 30.

2. Upon the Base *a c* of the given Triangle *a c b*, complete the Geometrical Square *a c i k*, and draw one of its Diagonals as will pass thro' any one Side of the given Triangle, as the Diagonal *c i*, which passes thro' the Side *a b* in *x*.

3. Draw the prick'd right Line *l a* parallel to *a c*, and to pass thro' the angular Point of the Triangle *b*.

4. From the angular Point *b*, draw the right Line *b n* parallel to *c k*, as also the right Line *r q* from the Point *r*, where the parallel right Line *l a*, intersects the Diagonal *c i*.

5. Lay a Ruler from the Point of Sight *e* to *q*, and draw up the Radial *q s*, until it meet the Diagonal *a h* in *s*. Thro' this last produc'd Point *s* draw the right Line *p m*, parallel to the traverse Line *h g*, then will the right Line *p m* represent the right Line *l a*.

Now to find the Perspective Point answerable to the angular Point *b*, lay a Ruler from *e* the Point of Sight to the Point *n* in the ground Line, and it will cut the Line *p m* in *z*, the Point requir'd.

Lastly, Draw the right Lines *g z*, and *z h*, and the Triangle *g h z* will be the Perspective Appearance of the given scalenum Triangle *a b c*, being seen in a direct View as requir'd.

Secondly, For the oblique View.

Fig. 31.

1. Let the scalenum Triangle *a b c* be given, and let its Base *a c* represent the ground Line; also let *d* be the given Point of Sight, and *e f* the given Point of Distance; also let the Geometrical Square *a c i k* be describ'd on its Base *a c*, with the Diagonal *c i* and parallel right Lines *p l*, *n b*, and *b q* be drawn, as before in the direct View.

2. Draw the Radials *d a* and *d c*, also the Diagonals *f a* and *e c*, and complete the Perspective Square *g h a c*.

3. Lay

3. Lay a Ruler from the Point of Sight *d* to the Point *l*, and draw up the Radial *l n*, until it meet the Diagonal *a h* in *n*, and thro' the Point *n* draw the right Line *m l* parallel to the traverse Line *g h*.

4. In like manner, lay a Ruler from the Point of Sight *d* unto the Point *u*, and draw up the Radial *u o*, until it meet the Line *m l* in *o*, then shall this last produc'd Point *o* represent the angular Point *b* of the given Triangle *a b c*.

And if you draw the right Lines *g o* and *h o*, the Triangle *g h o* will represent the Perspective Appearance of the given Triangle *a b c*, being seen in an oblique View as requir'd.

☞ I advise the Learner to be perfect in this last Problem, before he proceeds further, and to find out the angular Point *o*, according to the preceding Rule at the End of the last Section, for by either of these Rules any given right Line or Point may be most easily and exactly represented, let the Position be ever so oblique.

PROBLEM IV.

A Geometrical Square being given, with its two opposite Angles in a right Line with the Eye, to find its Perspective Appearance, being seen either in a direct or oblique View.

First, For the direct View.

PRACTICE. Fig. 32.

1. Let the Geometrical Square *n o p r* be given, with its Diagonals *n r* and *o p*.

2. Draw the ground Line 4 3 thro' any Angle thereof as *n*, at right Angles to the Diagonals *n r*, and continue on *r n* to the Point of Sight assign'd at Pleasure, and place the Points of Distance *a* and *c* at Pleasure also.

3. Make *n l* and *n k*, equal to *m p* and *m o*, and complete the Geometrical Square *k l q s*, which will then circumscribe the given Square *n o p r*.

4. To the Square *k l q s*, draw down the Radials *b k* and *b l*, as also the Diagonals *a l* and *c k*, and complete the Perspective Square *d f k l*.

Seeing the Perspective Square *d f k l*, represents the Geometrical Square *k l g s*, and the given Square *n o p r* is inscrib'd in the Square *k l q s*; and since that the angular Points of the inscrib'd Square *n o p r* are represented in the Perspective Square by the Points *e g i n*; therefore drawing the right Lines *e g e i*, and *n g n i*, they will complete the Perspective Appearance *e g n i*, of the inscrib'd

scrib'd given Square *n o p r*, being seen in a direct View as requir'd.

Secondly, For the side or oblique View.

1. Let the given Square be *l o p r*, circumscrib'd as before, with the Geometrical Square *k m q s*, and let *b* be the Point of Sight, and *a c* the Points of Distance given at Pleasure.

2. Draw the Radials *b k*, *b l*, *b m*, as also the Diagonals *a m* and *c k*, and complete the Perspective Square *d f k m*.

3. Draw the Diametrical *h g* parallel to the Traverse *d f*, and thro' the Intersection *i* of the Diagonals *k c* and *k m*. Since the angular Points *o l*, *p r*, in the Geometrical Square *k m q s*, are represented in the Perspective Square, by the Points *h e l g*; therefore draw the right Lines *e h*, *e g*, and *h l*, *l g*, they will complete the Perspective Appearance *e h g l*, of the inscrib'd given Square *l o p r*, being seen in an oblique View as requir'd.

☞ *Note*, These inscrib'd Squares might have been found in another manner as following. Continue the Sides *r p* and *r o* of the given Square *o n r p*. (*Fig.* 32.) up to the ground Line 4 3, and then drawing the right Lines *a* 4, and *c* 3, the Parts *e g* and *e i* will be two Sides of the requir'd Square. *Lastly*, If from the Points *g* and *i*, (where the Lines 3 *c*, and *a* 4 intersect the Radials *b k* and *b l*) you draw right Lines to the Point *n*, they will complete the Perspective Square *e g n i* requir'd.

Fig. 33. *Note*, Also the same is to be understood in the oblique View, where the Sides *r p* and *r o*, are continu'd on to the ground Line in Points 1 and 2; and from thence the Lines 1 *c*, and *a* 2 being drawn, whose Parts *e g* and *h e*, limit the upper ½ of the Square, and the lower ½ by the Lines *l h* and *l g*, drawn from their Intersections *h g* of the Radials *b m* and *b k*, to the Point *l*, as before.

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PROBLEM V.

Divers concentrick Geometrical Squares being given, with the Points of Sight and Distance also, to find their Perspective Appearance, being seen either in a direct or oblique View.

First, For the direct View.

1. Let the Geometrical *f i g h*, with the interior or concentrick Squares *k m p q*, and *l n o r* be given, as also the Points of Sight *b*, and Distance *a* and *c*.

2. Draw the Radials *b f* and *b i*, also the Diagonals *a i* and *e f*, and complete the the Perspective Square *d e f i*.

3. Con-


Fig XXII
Fig XXIII
Fig XXI
Fig XXVI
Fig XXV
Fig XXIIII
Fig XXVIII
Fig XXIX
Fig XXVII
Fig XXXII
Fig XXXI
Fig XXX

3. Continue the Sides *k p* and *m q*, of the Square *k m p q*, to the ground Line *f i* in the Points *s* and *x*; also continue on the Sides *l o* and *n r* of the Square *l n o r*, to the ground Line *f i*, in the Points *t* and *u*. Fig. 34.

4. Lay a Ruler from the Point of Sight *b*, to the several Points *s t u x*, and draw the Radials *x* 5 *u* 4, *t* 6, and *s* 3; then will their Parts, *viz.* *n* 5, 9, 4; 2 6; and 3, 1; represent the Sides *k p*, *l o*, *n r*, and *m q*. And if you draw the right Lines 5, 3; 4, 6; 9, 2; and *n* 1, they will complete the Perspective Appearance of the given Geometrical concentick Squares *k m p q*, and *l n o r*, being seen in a direct View as requir'd.

Secondly, For the oblique View.

1. Let the Squares *d p m o*; *e g l n*; and *f b i k* be given as before, with their Sides *e l f i*, *b k* and *g n*, continu'd up to the Points 2, 34, and 5, in the ground Line *dp*. Fig. 35.

2. Let the Point of Sight *t*, and Points of Distance *s u* be determin'd, and the Radials *t d*, and *t p*, and the Diagonals *s p*, and *u d* be drawn, as also the Traverse *b c*.

3. Lay a Ruler from the Point of Sight *t*, to the several Points produc'd by the Continuation of the Sides of the Squares in the ground Line, *viz.* the Points 2, 3, 4 and 5, and (as before) draw the Radials 2 *a*, 3 *g*, 4 *t*, and 5 *i*, whose Parts 6 *a*, 8 *g*, 9 *t*, and 7 *i*, represents the Sides *e l*, *f i*, *b k*, and *g n* of the given Squares *e g l n*, and *f b i k*.

Lastly, If you draw the right Lines *a i*, *g t*, 8, 9; and 6, 7; they will complete the Perspective Appearance of the given Squares, being seen in an oblique View as requir'd.

PROBLEM VI.

The same Geometrical Squares being given, with two of their opposite Angles, plac'd in a right Line with the Eye, to find their Perspective Appearance, in a direct and oblique View.

First, For the direct View.

PRACTICE. Fig. 36.

1. Let the Geometrical Square 2 *u y x*, with its interior or concentrick Squares *n m t s*, and *o p r q* be given, with their Diagonal 2 *m o r t x*, plac'd directly in a right Line with the Point of Sight F, which let be given at Pleasure, as also the Points of Distance E G.

2. Thro' the Points or Angles *u y*, draw the two right Lines *i z* and K *w*, parallel and equal to the Diagonal 2 *x l* H, which terminate in *w z*, and complete the Geometrical Square *c i w z*. Let *c i*

be the ground Line, and draw the Radials F L, and F K, also the Diagonals G K, and E L; likewise draw the direct Radial F 2, and Diametrical A D thro' the Intersection of the Diagonals.

3. Continue up the several Sides of the given Squares *x u y x*, *n m s t*, and *o p q r* to the ground Line *l* H at the Points *l k i g f e d c b a* and from the Points of Distance E and G, draw the right Line *a* G, cutting the Radial F K in A, and Traverse in 9, then will the Part A 9 represent the Side 2 *u* of the greatest given Square 2 *u y x*.

Fig: 36.

In the same manner draw *b* G, and the Part 6, 8; will represent the Side *n m* of the Square *n m s t*, also draw C G and the Part 5, 7; will also represent the Side *p o* of the little Square *o p r q*.

Proceed in the same manner to draw the Lines *d* G, *e* G, and E *l*, E *k*, E *i*, E *g*, and E *f*, and their Parts cut by the Diametrical D A, and direct Radial 9 *h*, which represents the two Diagonals 2 *x*, and *u y* at the Points 9 D, 8, 3; 7, 4; 5, 1; 6, 2; will represent the Sides 2 *y m s*, *o q*, *p r*, *u t* and *n x* of the given Squares, and thus will they be respectively terminated in a direct View as requir'd.

Secondly, For the oblique View.

Fig. 37.

1. Let the Geometrical Squares *d h s z*, *i q m l*, and *k p o n* be given, with the Geometrical Square *g r o t*, circumscrib'd as before in the direct View; also let *g o* be continu'd out at Pleasure, as to D B, and let it be the ground Line.

Continue up all the Sides of the several Squares (as in the preceding) unto the ground Line D B, in the Points B *a* 2 *y x* and *u w i g* D. Let F be the given Point of Sight, and E G the given Points of Distance, and draw the Radials F *g*, F *o*, and the Diagonals E *o* and G *g*, and complete the Perspective Square *e f g o*, which represents the great circumscribing Square *o g t r*.

2. Draw the Diametrical *a b* thro' the Point of Intersection of the Diagonals and direct Radial F *d*; for it is the Diametrical 6 *a*, and direct Radial *c d*, that represents the Diagonals *d s*, and *z h*, and limits the Representations of the Geometrical Squares given, as following.

3. Draw right Lines from the Points B *a* 2 *y x*, to the Point of Distance E, and their respective Parts will be terminated by the Radial *c d*, and Diametrical 6 *a*, as in the direct View preceding:) Then performing the like Operation from the Points *u w i g* D to the other Point of Distance G their respective Parts will be terminated as before, and complete the Perspective Squares being seen in an oblique View as requir'd.

PRO-

PROBLEM VII.

Divers Geometrical Squares inscrib'd within each other, with the Points of Sight and Distance given, to find their Perspective Appearance in direct and oblique Views.

First, For the direct View.

1. Let the inscrib'd Squares *7 s t q*; *e h o z*, *f r g i*, and the circumscribing Square 5, 6, *p r*, with the Points of Sight B, and Distance C A be given, and let the Radials B 5, B 6, and the Diagonals A 6, and C 5 be drawn, and complete the Perspective Square 3, 2, 5 6. Fig. 38.

2. Draw the Diametrical 5, 4; and C 1 thro' the Intersection of the two Radials B 6, and B 5, with the Lines C 7, and A 7. also the direct Ray B 7.

This done, draw 1 *a*; 1, 7; 4, *a*; 4, 7; and they'll complete the Perspective Appearance or Square 1 *a* 4 7, which represents the Geometrical Square *7 s t q*.

3. Draw *d* 8, *d c*, *c* 9, and 8 9; and they'll complete the Perspective Appearance or Square *d c* 8 9; which represents the Geometrical Square *e h o z*.

4. Draw *t e*, *e* 1, 1 *n*, and *n t*, and they'll complete the interior Perspective Square *t e n* I which represents the Geometrical Square *f r i g*.

Lastly, The Geometrical circumscribin Square is represented by the Perspective Square 2, 3, 5 6, and thus do these inscrib'd Squares appear, being seen in a direct View as requir'd.

Secondly, For the oblique View.

1. Let the Geometrical Squares *g f w y*, *e u z x*, *k l t s*, and *i n p r* be given as before, and let B be the given Point of Sight, and A C the given Points of Distance, and draw the Radials B *g*, B *e*, B *f*, and Diagonals A *f*, C *g*, and proceed as follows. Fig. 39.

Draw the Lines *e* C, *e* A, and *c* C, *d* A and they complete the Perspective Square 7 *c d e*, which represents the Geometrical Square *e u z x*. *Again*, Draw the right Lines 4, 6; 4, 3; 3, 2; 6, 2; and they complete the Perspective Square 4, 6, 3, 2; which represents the Geometrical Square *k l t s*.

Lastly, Draw the Lines 8, 4, 4 1; 1 *x*; and 8 *x*, and they'll complete the Perspective Square 8 4, 1 *x*, which represents the Geometrical Square *i n r p*, and the Perspective Square *b* 8 *g f*, the Geometrical circumscribing Square *g f w y*. And thus do the given inscrib'd Squares appear, when seen in an oblique View as requir'd.

PROBLEM VIII.

The preceding Geometrical Squares being given, with two opposite Angles of the circumscribing Square, plac'd in a right Line with the Point of Sight, to find the Perspective Appearance, being view'd in direct and oblique Views, the Points of Sight and Distance being given as before.

First, For the direct View.

PRACTICE.

Let the Geometrical Square *i* 4, 1 *y* be plac'd with its opposite
Fig. 40. Angles *i y* in a right Line with the Point of Sight 3, and let the Squares *o q w u*, *p r s t* be inscrib'd, and let *l n* be drawn thro' *i*, at right Angles to the Line *i y*, representing the ground Line. Make *i m* and *i k*, each equal to 4 *x*, or *x* 1, and complete the Geometrical Square *k m x z*, and to it draw the Radials 3 *k*, 3 *m*, and Diagonals 4 *k*, and 2 *m*, (the Points of Sight and Distance being first determin'd at Pleasure) and complete the Perspective Square *d o k m*, which represents the Geometrical Square *k m x z*.

2. Draw the Radial 3 *i*, and the Diametricals 4 *i*, 2 *i*.

3. Draw the right Lines 6, 7, 4; and they will complete the Perspective Square *h* 6 7 *i*, which represents the Square *i* 4, 1 *y*. *Again*, Draw the Lines *e f*; *e g*; *h f h g*; and they'll represent the Geometrical Square *o q w u*.

4. Draw the Lines *c n*, *c a*, *a m*, *m n*, and they'll complete the Perspective Square *m n c a*, which represents the Geometrical Square *p r s t*, and thus do the inscrib'd Squares appear, being seen at an Angle in a direct View as requir'd.

Secondly, For the oblique View.

1. Let the Squares *i h x z*, *n s r y*, *m o t w*, and *n p u q* be given as those of the direct View, and let the Point of Sight 2, and
Fig. 41. Points of Distance 1 3, be given at Pleasure, and let the Radials 2 *h*, 2 *i*, and Diagonals 3 *h* and 1 *i* be drawn, and complete the Perspective Square 5 4, *h i*, which will represent the Geometrical Square *h i x z*. Also draw the Radial 6 *n* from the Point of Sight, thro' the Intersections of the Diagonals to the ground Line at *n*, and the Diametrical *g* 8, thro' the said Point of Intersection also.

2. Continue up the Sides *y r* and *y s*, to the ground Line *k* and *l*, and then laying a Ruler from the Point of Sight 3, to the Points *l* and *n*, draw the right Line *g* 6, and 8 *n*; also from the other Point of Distance 1, to the Points *n k*, draw the right Lines 8 6, and *g n*, which

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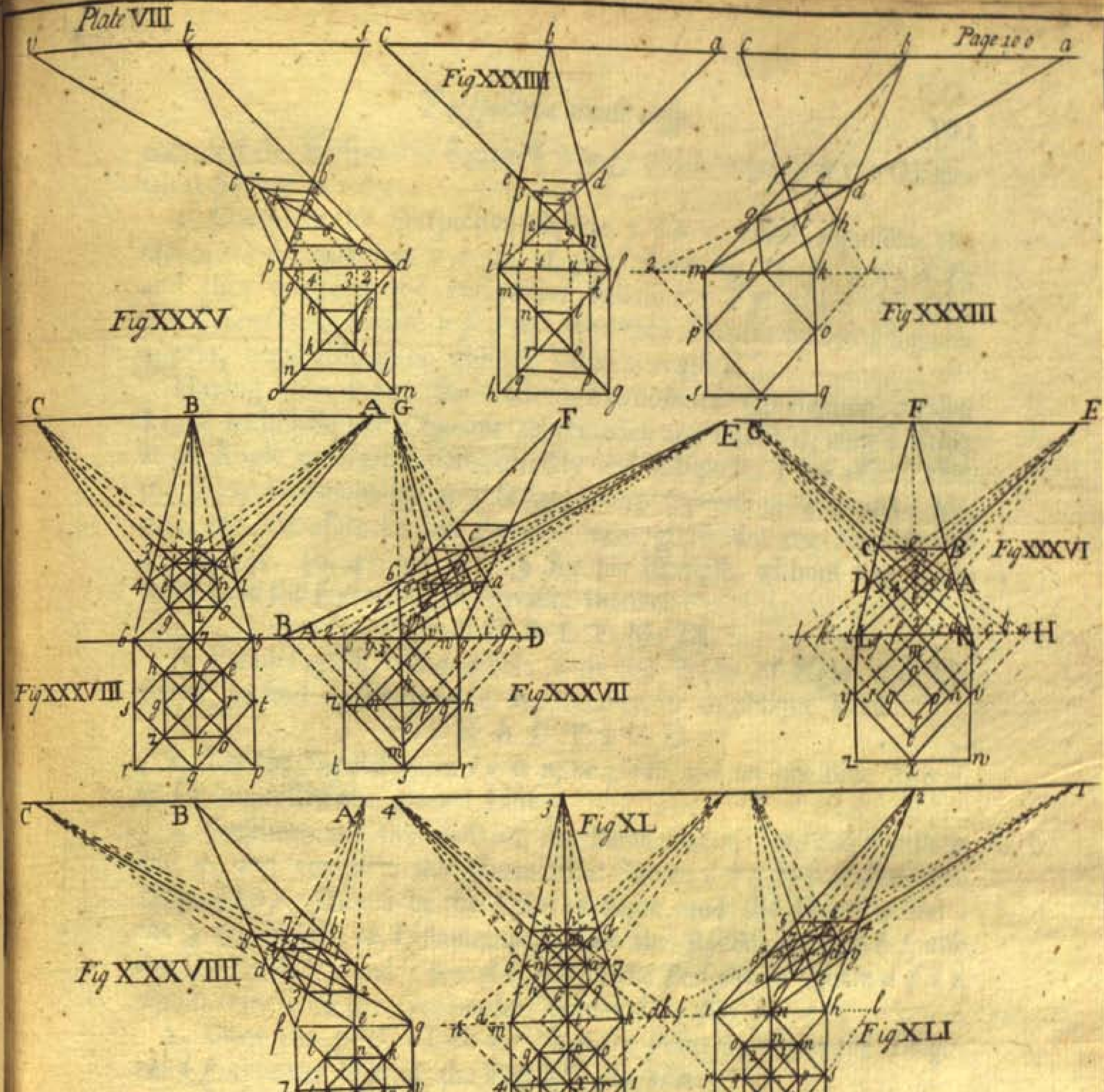


Fig XXXIIII
Fig XXXV
Fig XXXIII
Fig XXXVI
Fig XXXVIII
Fig XXXVII
Fig XXXVIIII
Fig XL
Fig XLI
Direct of ye View Side
Oblique of ye View Side
Direct of ye View Side
Fig XLII.
Fig XLIIII
Fig XLIII

complets the Perspective Square 8 6 *n g*, which represents the Geometrical Square *n s r y*.

3. Complete the Perspective Square 7 6 *a e*, which represents the Geometrical Square *m o t w*. *Lastly*, Draw *e g*; *g f*; *e d*; *d f*; and they complete the Perspective Square *e g d f*, which represents the Geometrical Square *n p q u*. And thus do these inscrib'd Squares appear, being seen in an oblique View as requir'd.

Having thus shewn the various Methods of representing parallel Lines within Squares, Squares concenterick and inscrib'd, view'd either at an Angle or Front, both directly and obliquely, I am assur'd that the Learner cannot be at a Stand in any Operations of the like Nature, and therefore for his Exercise have given him the *Figures* 42, 43, 44, 45, 46, 47, 48 and 49 for his Exercise, without any thing more than the several Lines requisite thereto.

PROBLEM IX.

A Parallelogram being given, with the Points of Sight and Distance, to find its Perspective Appearance, in an oblique View.

PRACTICE.

1. Let the Parallelogram *l o m n*, be given, and let one Side thereof, as *l o*, represent the ground Line.

2. Continue out the Ends of the Parallelogram *l m*, and *o n* to *p*, and *q*, and complete the Geometrical Square *l o q p*, and draw the Diagonal *l p*. Let *b* be the Point of Sight, and the Points *a* and *c* the given Points of Distance, and draw the Radials *b o* and *b l*, also the Diagonals *a l* and *c o*, and complete the Perspective Square *d f l o*, which represents the Geometrical Square *l o q p*. *Fig.* 5.

3. Draw the prick'd Line *k r* from the Point *k*, where the Diagonal *l p* cuts the Side of the Parallelogram *m n*.

4. Lay a Ruler from *b* to *k*, and it will cut the Diagonal *o f*, (which represents *p l*) in *i*, then will the Point *i* represent the Point *r*, and if thro' the Point *i* you draw the right Line, or Diametrical *e g*, it will complete the Perspective Parallelogram *d f e g*, which represents the Geometrical Parallelogram *l o m n*, being seen in an oblique View as requir'd.

☞ But if the Parallelogram *l o m n* had been plac'd at *u t q p*, then its Perspective Appearance in this View would would have been the Perspective Parallelogram *u t i i k l*, which is thus found.

Where the Side of the Parallelogram *x x* intersects the Diagonal in the Point *s*, draw up the right Line *s h* parallel to *o p*; then lay-

ing

ing a Ruler from *b*, the Point of Sight, to *b* in the ground Line *l o*, it will cut the Diagonal *c o* in *x*; and if the Diametrical or right Line *u t* be drawn thro' the Point *s* it will complete the Perspective Parallelogram *z z l o*, which is the Appearance of the Parallelogram *u t q p*, being seen in oblique View as requir'd.

☞ Now seeing that there's the very same Lines us'd in every of the oblique Views, in direct Views, which has been sufficiently shewn in the several Examples hereof, I shall therefore for the future omit the Explanation of the oblique Views, and give their Representations with their respective Lines only; for Repetitions now are rather injurious than instructive to the young Learner, who by this time must be tolerable well inform'd of the Nature of superficial Perspective.

PROBLEM X.

The preceding Parallelogram being given with the Points of Sight and Distance, to find its Perspective Appearance in a direct and oblique View to one of the Ends thereof.

For the direct View.

PRACTICE.

1. Let the Parallogram *k h l o* be given, and let one End thereof as *k h* be plac'd in the ground Line *g i*. with its centeral Line *z x* directly opposite to the Point of Sight *b*, which is supposed to be at Pleasure, as also the Points of Distance *a* and *c*.

Fig. 52: 2. Make *i k h g*, each equal to ½ one of [the longest Sides of the Parallelogram, as *h l*, or *k o*, and complete the Geometrical Square *i g m k*, and draw the Radials *b g*, *b i*, and Diagonals *c g*, *a i*, and complete the Perspective Square *d f g i*, which represents the Geometrical Square *g. i k m*.

3. Lay a Ruler from the Point of Sight *b*, to the two Angles of the Parallelogram *k h*, and draw the Radials *e h* and *i k*, which will complete the Perspective Parallelogram *e i k h*, which is is the Perspective Appearance of the Geometrical Parallelogram *k h o l*, been seen in a direct View as requir'd.

Fig. 53 is the same Parallelogram seen in an oblique View as there represented.

PROBLEM XI.

A Pentagon being given, with the Points of Sight and Distance, to find its Perspective Appearance in a direct View with one Side in front, cutting the principal Ray at right Angles.

1. Let

1. Let the given Polygon *Fig.* 56 be *q o s w y*, and let *o s q x y* represent the ground Line. Also let the Point *b* be the given Point of Sight, and *c a*, the given Points of Distance.

3. Complete the Geometrical Square *m p y t*, with its Diagonal *m t*, and its Sides will be equal to *s q*, and draw the Radials *b m*, and *b p*, and the Diagonals *c p* and *a m*; and complete the Perspective Squares *f c m p*: and thus are we prepared for the Work in hand as follows. *Fig.* 56.

4 Draw the Line *s q*, and from the Point *r*, where the Line *s q* cuts the *Diagonal m t*, draw up the right Line *r k* parallel to *m s*.

5. Lay a Ruler from the Point of Sight *b*, to the Point *k*, and where it meets the Diagonal *c p* in *z*, draw the Diametrical *i h g*.

6. Draw the right Lines *d i*, *d g*, *i l*, *g n*, and they complete the Perspective Pentagon *d i g l n*, which is the Perspective Appearance of the Geometrical Pentagon *o s q x y* as required. *Fig.* 55 is the same Pentagon as it appears in a oblique View.

PROBLEM XII.

The same Pentagon being given, with the Points of Sight and Distance, to find the Perspective Appearance in a direct View, with one of its Angles plac'd in a right Line with the Eye and Point of Sight.

PRACTICE.

1. Let *o q s y* be the given Pentagon with its Angle *y* plac'd opposite to the Point of Sight *b*, and let *z x* be plac'd as to be bisected in *y*.

Inscribe the Pentagon in the Parallelogram *m r z x*, and make *m r* the ground Line. *Fig.* 54.

2. Let B be the given Point of Sight, and A C the Points of Distance and draw the Radials B *m*, B *r* and Diagonals A *m* and C *r*, and complete the Perspective Square *c h m r*. (3.) Draw the right Line *w s*, and from the Point 1, where 'tis intersected by the Diagonal *m x*, draw the right Line 1 2 parallel to *m z*. (4.) Lay a Ruler from *b* to 2, and it will cut the Diagonal *c r* in *h*, thro' which Point draw the Diametrical *l i*, then will *l i* represent *w s* in the Geometrical Square *m r z x*.

5. Draw the Radials *d o*, *b q*, cutting the Traverse *e h* in *d* and *b*. Then drawing the right Lines *d l*, *l p*, *p i*, *i b*, and they will complete the Perspective Pentagon *d b l i p*, which is the Appearance of the Geometrical Pentagon *o q w s y*, being seen at the Angle *y* in a direct

direct View as requir'd. *Fig.* 57, is an oblique View of the same Pentagon seen at an Angle as before.

PROBLEM XIII.

A Hexagon being given, with the Points of Sight and Distance, to find its Perspective Appearance, being seen in a direct View with one side in front cutting the principal Ray at right Angles.

PRACTICE.

1. Let *l m p q s r. Fig.* 58. be a given Hexagon, and let one Side thereof as *l m*, represent the ground Line, continu'd both ways to *n* and *o*, making *o n* equal to *q p*, and then completing the Parallelogram *n o t u*. draw the Diameters *p q x a*, and the right Lines *s l s* and *m r*.

2. Let the Points *a b c* be the given Points of Sight and Distance and draw the Radials *b n b o*, and Diagonals *c n*, and *a o*, and complete the Perspective Square *d g n o*.

3. Lay a Ruler from *b to l* and *m*, and draw *e l h m*, then will *e h* represent *l r*. Draw *i k* parallel to *d g*, thro' the Point of Intersection of the Diagonals A cutting *d n* in *i*, and *g o* in *k*.

4. Draw the right Lines *e i*, *i l*, *h k*, and *k m*, and they complete the Perspective Hexagon as requir'd. *Fig.* 59, is an oblique View of the same.

PROBLEM XIV.

The same Hexagon being given with the Points of Sight and Distance, (as before) to find its Perspective Appearance being seen in a direct View at an Angle opposite to the direct Ray of Sight, as *Fig.* 60.

PRACTICE.

1. Let the given Hexagon be *r x u* 3 *i* 6 plac'd with one of its Angles as *r* in the ground Line *p q*, and its Sides *u i* and *x* 3, perpendicular to the same. Continue *i u* to the ground Line in *p*, as also 3 *x* to *q*. Likewise *x* 3 to 4, and *u i* to 8, making *i* 8 equal to *u p*, and 3 4, equal to *q x*; then will the Line 8 4, pass by the Angle 1 6 3, and draw one of the Diagonals of the Parallelogram, as *q* 8.

2. Draw the right Line *u x*, and *i* 3; cutting the Diagonal *q* 8, in the Points *w z*, from which draw up the right Lines *w t*, and *z s*, to the ground Line *p q*, and parallel to the Side *p* 8.

3. Let the given Point of Sight be *b*, and Points of Distance *a* and *c*; also let the Radials *b p*, *b r*, and *b q* be drawn, and the Diagonals *c p* and *a q*, and complete the Perspective Parallelogram *d e p q*, which represents the Parallelogram *p q* 8 4.

4. Lay


Fig. 46.
Fig. 47.
Fig. 45
Fig. 50
Fig. 49.
Fig. 48
Fig. 52
Fig. 51.
Oblique View
Fig. 53
Oblique View
Fig. 56
Fig. 55.
Fig 54

Lay a Ruler from the Point of Sight *b* to *t*, and it will cut the Diagonal *p c* in *o*, thro' which Point draw the right Line *g h* parallel to *d e*, cutting the Radials *d p* in *g*, and *e g* in *h*.

5. Lay a Ruler from the Point of Sight *b* to *s*, and it will cut the Diagonal *p c* in *i*, thro' which draw the right Line *l m*, cutting *d p* in *l*, and *e q* in *m*.

6. Draw the right Lines *f g*, *f h*, *r l*, *r m*, and they complete the Perspective Hexagon *f d h l r m*, as requir'd. *Fig.* 61 is an oblique View of the same Figure.

PROBLEM XV.

A Septagon being given, with one of the Angles placed directly before the Eye, to find the Perspective Appearance, the Points of Sight and Distance being given. *Fig.* 62.

1. Let the given Septagon be *r y x*, 4, 6, 7 8; with one of its Angles, (as *r*) placed in the ground Line *n u*, and its Diameter *r* A perpendicular thereto.

2. Make *r u*, and *r n*, each equal to *h* 6, or *h* 4, and continue 7, 8, to 9 B, making A B, A 9, equal to *r u*, *r n*, and draw *n* 9, *u* B, which completes the Parallelogram *n u* 9 B.

2. Draw the Diagonal *u* 9, and the right Lines *y x* 4, 6; cutting the Side *n g* in *z* and 4; the Side *u B* in *x* and 6; and the Diagonal *u* 9 in *n* and 5; and draw up the Parallel Lines 5 *p*, *n s*, *i t* and 7, *o*; from the Points 7, 5, *n* and *i*.

3. Let D be the given Point of Sight, and C E the given Points of Distance, and draw the Radials D *n*, D *u*, and Diagonals C *u*, and E *n*, and complete the Perspective Parallelogram *g f n u*.

4. Lay a Ruler from D to *s*, and it will cut the Diagonal *n* E in *d*, thro' which Point draw the right Line *b e* parallel to the Traverse *g f*. Then lay a Ruler from D to the Point *t*, and it will cut the Diagonal *n* E in *c*; make *z a* equal to *z c*, and draw the Lines *h a*, *h c*, and they will represent *r y*, *r n*, in the Geometrical Parallelogram *n n* 9 B.

5. Lay a Ruler from the Point *p* in the ground Line *n u* to the Point of Sight D, and it will cut the Diagonal *n* E in *k*, thro' which draw the Line *l k i* parallel to the ground Line *n u*, (then will the Point *l* and *i* represent the Points 4 and 6) and draw the Lines *a l* and *c i*, which represent the Lines *y* 4 and *i* 6.

6. Draw the Lines *l o*, and *i q*, and the *Figure h c i q o l a* will be the Perspective Septagon required. *Fig.* 63 is an oblique View of the same *Figure*.

PROBLEM XVI.

The same Septagon being given, with one of its Sides placed at right Angles to the direct Ray, to find the Perspective Appearance thereof, the Points of Sight and Distance being given.

PRACTICE.

Fig. 64. Let the Septagon *q p*, 8, 5, 4, *z x*, be given, with one of its Sides (as *q p*) placed in the ground Line *w* D.

2. Draw the Diameter *h* 4; and the right Lines D 6 and *w* 1, parallel thereto, at the Distance of *d* 10, and *d x*; and complete the Parallelogram *w* D, 1 6, and draw its Diagonal D 1.

3. Draw the Lines *x* 10, and *z* 5, cutting the Diagonal in the Points 9 and *y*. Then from the Points 9 *y z*, draw up the Lines 9 *p y s*, *z l* parallel to *h* 4, which terminate in the ground Line at *p h s*.

4. Let B be the Point of Sight, and A C the given Points of Distance, and draw the Radials B *w*, B D, and Diagonals A D, and C *w*, and complete the Perspective Parallelogram *a d w* D.

5. Lay a Ruler from the Point of Sight B, to the Point *p* in the ground Line, and it will cut the Diagonal *w* C in *e*, thro' which Point draw the right Line *g n*, cutting the Radials B *w* in *g*, and B D in *n*.

6. Lay a Ruler from the Point of Sight B, to the Points *s q*, and it will cut the Traverse *a d* in *b* and *c*, then drawing the Lines *g b*, and *c n*, they being taken with *b c*, will represent the Sides *x q*, *q p*, and *p* 10, of the given Septagon, *q*, *p*, 10, 5, 4, *z*, *x*.

7. Lay a Ruler from the Point of Sight B, to the Point *s*, in the ground Line *w* D, and it will cut the Diagonal *w* C in *k*, thro' which Point draw the right Line *l m*, also from *l* in the ground Line to B, lay a Ruler and it will cut *l m* in *n*, and make *f i* equal to *f n*.

Lastly, Draw the Lines *n g*, *i n*, *n h*, *h i*, and they complete the Perspective Septagon as requir'd. *Fig.* 65 is an oblique View of the same.

PROBLEM XVII.

An Octagon being given, with one of its Sides plac'd in the ground Line, to find the Perspective Appearance thereof, in an oblique View, the Points of Sight and Distance being given.

PRACTICE.

Fig. 69. 1. Let the Octagon *m o t w* 3, 2, *u q*, be given, with one of its Sides placed on the ground Line *l p*, with its Diameter *n z* perpendicular thereto.

2. Make

2. Make *z p*; *z l*; *n* 4; *n* 1, each equal to ½ *z n*, the Semi-diameter, and complete the Geometrical Square *l p* 14, and draw its Diagonal *l* 1, also the right Lines *q t*, *u w*.

3. Let the Points 6, 5, 7, be the given Points of Sight and Distance, and let the Radials 6 *l*, 6 *p*, and the Diagonals 7 *l*, 5 *p* be drawn, and the Perspective Square 8, 9, *l p* be completed.

4. Lay a Ruler from the Point of Sight 6, to the Points *w t*, in the ground Line, and it will cut the Traverse 8, 9, in *a* and *b*, and the Diagonals in *d c*, *i k*.

5. Draw the right Line *c f*, thro' *d* and *e*; and *g h*, thro' *i* and *k*, then will the Points *a b f h t w g e* be the angular Points, and if the right Lines *r c*, *b f*, *h t*, and *g w* are drawn, they will complete the Perspective Octagon or Appearance of the Geometrical Octagon given, as requir'd.

PROBLEM XVIII.

The same Octagon being given, with one of its Angles in a direct View to the Eye, to find its Perspective Appearance, the Points of Sight and Distance being given.

PRACTICE.

1. Let the given Octagon be *i n m l q p o l m*, inscrib'd in the Geometrical Square *h k w s*, and then making one Side of the Square the ground Line (as *h k*) the Angle *i* will stand in a direct View as given. *Fig.* 67.

2. Draw the Diagonals *h s*, and *k w*, and from the Points *n p*, and *m o*, draw the Lines *p r* and *m o*, terminating in the ground Line, at *r o*.

3. Lay a Ruler from the Point of Sight *y* to the Points *r o*, and it will cut the two Diagonals in *r a*, *n a*; then drawing *f g* parallel to the Traverse thro' the Intersection of the two Diagonals at B, it will cut the Radials *y h* in *f*, and *y k* in *g*.

Lastly, The right Lines *z a*; *a f*; *f a*; *a i*; *i n*; *n g*; *g r*; and *r z*. being drawn, they will complete the Perspective Octagon as requir'd.

PROBLEM XIX.

A Circle being given to represent its Perspective Appearance, having the Points of Sight and Distance determin'd.

Circles are usually describ'd upon a given Center, by the Help of a pair of Compasses, Line, &c. when represented in Plano, *viz.* Geometrically, as A F O K. *Fig.* 70. But when they are to be represented Perspectively, as *a f o k* (in the upper Part of the same *Figure*) then they cannot be perform'd in that manner, but by a Number of certain

certain Points, which terminates the Circumference thereof, as will now be exhibited in this Problem.

But, before I proceed thereto, it will not be amiss, if I should, in the first Place, shew how any Circle may be very easily describ'd, having the Diameter given, without any Regard being had to the Center, as follows.

Let it be required to describe a Circle, whose Diameter shall be equal to A B, *without having any Recourse to its Center.*

1. Make a Geometrical Square (as *l* 5 D E) with each of its Sides equal to the given Diameter.

2. Divide each Side thereof into any Number of even equal Parts, as 4, 6, 8, 10, 12, 14, 16, &c. In this Example I have divided each Side thereof in 8 equal Parts, *viz.* The Side *l* D, at the Points *a b c*, P *d e f l*; the Side *l* 5, in the Points *o s q*, A 2 3 4; the Side 5 E in the Points *o*, *g*, *q*, B, 2, 3, 4. *n k l* Q *i h g* E, and the Side D E in the Points *o p q* B 2 3 4.

3, Distinguish the Middle of each Side, as the Points A P Q and B, by particular Marks or Letters, or rather by drawing the two Diameters A B and P Q, which will divide the Square into 4 equal Parts.

4. Draw the right Lines P *o*, *d s*, *e q*, and *f* A, and their Intersections will generate or form one fourth Part of the Circle requir'd; then going thro' the like Operation with the other three Parts of the Square, you will have completed the whole Circle, without any Recourse to a Center as required.

☞ *Note*, That the more your Divisions are in Number, the exacter your Work will be.

By observing this Method of describing Arches, you will be led into a very easy Method of delineating the same *Figures* Perspectively, as following,

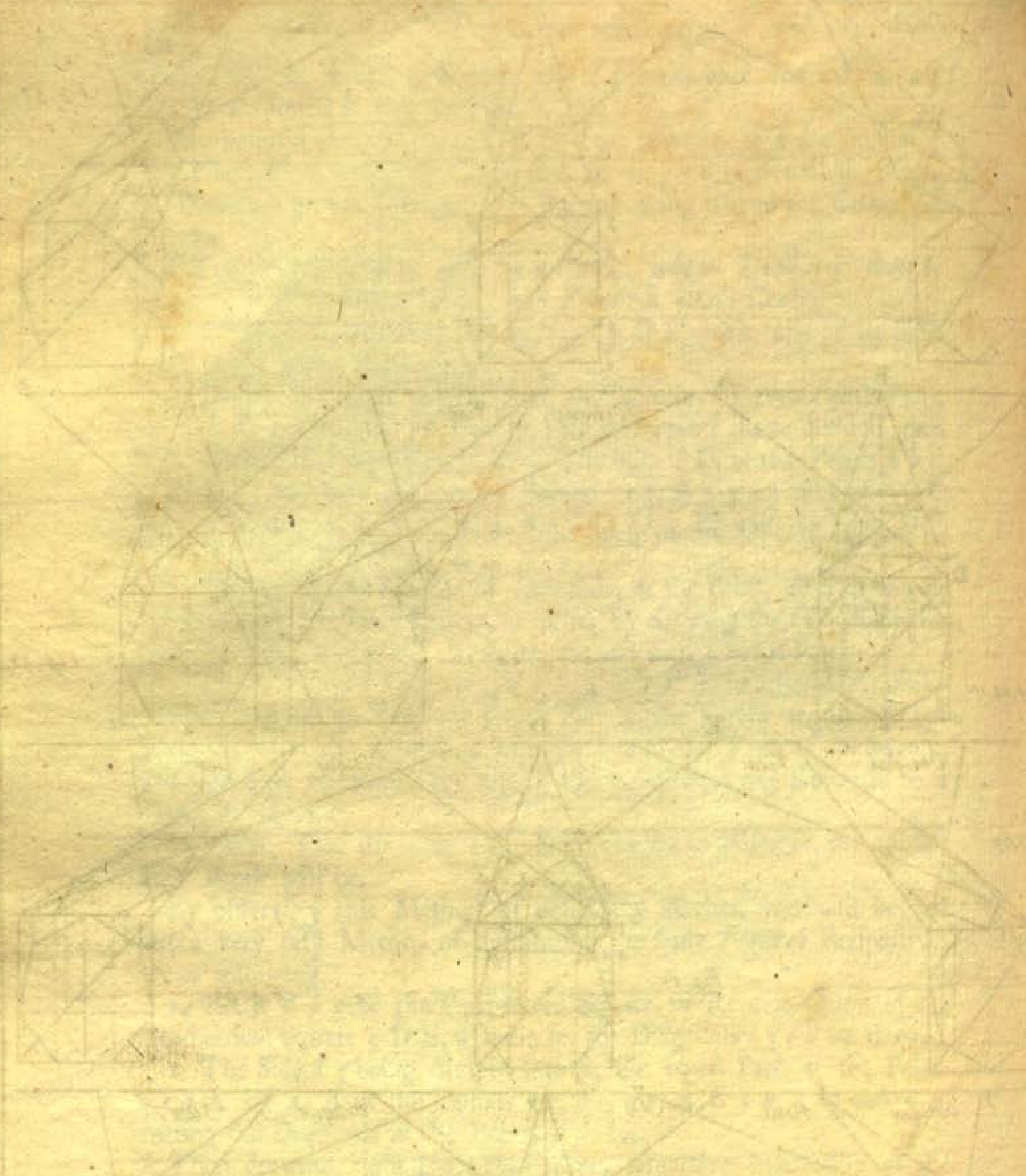
1, Let *e d l* 5 be the Perspective Square, or Representation of the Geometrical Square 5 D E, wherein let the Diagonals *e* 5 *l d* be drawn.

2. The Side *l* 5 being divided into 8, &c. equal Parts at the Points *o s q* A 2 3 4, draw the Radials *o r*, *s s*, *q t*, A C 2 *u*, 3 *w* and *y x*, cutting the Diagonals in the Points *x x*, &c.

Then drawing right Lines thro' every respective Intersection of the Diagonals, as *f c n b*, *m a* B A, *g* 7, *h* 6, *k* 8; they will divide the Sides *e l* and *d* 5, each into 8 equal Parts perspectively, at the Points *k h g* B *m n f*, and *c b a* A 6 8 5.

And since that a great Number of Lines do sometimes confound young Beginners, therefore the aforesaid Points of the four Sides of the Per-


Oblique View
Fig. 61.
Fig. 60.
Fig. 57.
Oblique View
Fig. 64.
Fig. 59.
Fig. 58
Oblique View
Fig. 65.
Fig. 62
Fig. 63.
Oblique View
Fig. 69.
Fig. 67
Fig. 68



Perſpective Square *e d l* 5, are repreſented by the Perſpective Square D E *e d*, to which the very ſame Letters are fix'd to the Diviſion of each Side reſpectively, but the Radials and Diametricals by which they were determin'd are left out, that the Effect of theſe interſecting Lines may the more plainer appear.

This being underſtood, begin, as in the Geometrical Circle, and draw the right Lines B *k*, 2 *h*; 3 *g*; 4 B; which will complete ¼ of the Perſpective Circle B B A C. *Again*, Draw the right Lines A *x*, *a w*, *b u*, and *c* C, alſo C *f t n*, *s m*, and *r* B. *Laſtly*, Draw B 3, *q* 7, *p* 6, and *o* A, and thus will you have deſcrib'd the Perſpective Circle B B A C, which is the Appearance of the Geometrical Circle A P Q B, being ſeen with the Points of Sight H and Diſtance I A as required. The other Circle in the Perſpective Square 5 K E L is the very ſame Circle, having its Points of Sight and Diſtance plac'd further from the ground Line 5 E than H I from the ground Line D E.

Here note, That if you divide the Side of the oblique Square X Z D *l*, in the ſame Manner, and proceed as herein deliver'd, the Interſections of the ſeveral Lines will truly repreſent the Circle P B A Q in that oblique View. And ſince that it will very often occur in Practice, as when you are to repreſent Circles in Plans of Gardens, Buildings, &c. therefore regard this well, and be well acquainted therewith, for 'tis of admirable Uſe, as hereafter will appear.

Now, altho' Circles may be thus perſpectively repreſented by Interſections of right Lines, and might be a ſufficient Direction without fur-Rules, yet, as this Method is entirely new, I will therefore alſo exhibit the old Method us'd by moſt Maſters, as following. *Fig.* 70.

A Circle being given, with the Points of Sight and Diſtance aſſign'd to repreſent the ſame perſpectively in a direct View.

1. Let the given Circle be K A F O, inſcrib'd within the Geometrical Square 1, 2, 3, 4.

2. Continue on the Side 4 K 2, to 4 in *Fig.* 73, and 3 F 1, to B 3, making 2 4 equal to 2 K, and 1 B to 1 F, and draw the right Line B 3, which will be parallel to the Side of the Square 1 2.

3. Biſect this laſt produc'd Line in *o*, and on *o*, with the Radius *o z*, deſcribe the Semi-circle 4 Z L, and divide the Arch thereof into any Number of even equal Parts, as 8, 10, 12, 14, 16, &c. In this Example I have divided it into eight equal Parts, at the Points *y x w z u t s*.

4. From theſe laſt divided Points *y x w z u t s*, draw right Lines to the right Line L 4, (which repreſents the ground Line) parallel to

o z.

o z. This done, and the Points of Sight and Distance being determin'd at A B D, draw the Radials B 4, B L, and the Diagonals A 4, D L, and complete the Perspective Square 1, 2, 4 L.

5. Lay a Ruler from the Point of Sight B, to the several Points E *a d o g f* and L in the ground Line L 4, and draw up the Radials E *b i*, *a q m*, *d l n*, *o a*, *g p c*, *f h a*, and L *g e*.

6. Through all the Intersections of these last drawn Radials, with the Diagonals, draw all the respective Diametricals, and their Points of Intersection *a c a e f g h p o l q b k i m n*, will be the several Points thro' which the Curve of the Circle must pass, being trac'd with an even Hand, so as not to make any Angles therein; and thus is the Perspective Appearance of the Geometrical Circle A K F O completed, after the old Method of *Sebastian Serlio*, and other eminent Masters of Perspective.

PROBLEM XX.

Two concentrick Circles being given, with the Points of Sight and Distance, to find the Perspective Appearance thereof.

PRACTICE.

Fig. 72. 1. Let the given Circles be those of *Fig.* 70, and let the external Circle be inscrib'd in the Geometrical Square 1 2 3 4.

2. Continue 1 3 to *q*, and 2 4 to *r*, making 3 *q* equal to F 3, and 4 *r* equal to K 4, and then drawing *q r*, bisect it in *b*, whereon, with the Distance B P describe the Semi-circle *q* P *r*, as in the preceding Problem.

3. Because the given Distance of the parallel Circles, is *x o*, therefore make P D equal thereto, and on *b* with the Radius *b s*, describe the Semi-circle *a p s*, and divide its Circumference into any Number of even equal Parts, as 8, 10, 12, 14, 16, &c. In this Example I have divided it but into 8, at the Points A B C D E F G.

4. From the Points A B C D E F G, draw right Lines parallel to D *b*, cutting *q r*, (which represents the ground Line) in the Points *y z x u w a*. Then having the Point of Sight B, and Distance A C, determin'd, draw the Radials B *q*, B *r*, and Diagonals A *q* C *r*, and complete the Perspective Square *a l q r*.

5. Lay a Ruler from the Point of Sight B, to the several Points in the ground Line *a y z x b u w a s*, and draw the Radials *a b*, *y c*, *z k*, *x d*, *b e*, *u f*, *w g*, *a i*, and *s h*; and where these Radials intersect the Diagonals, draw diametrical Lines (represented in the *Figure* by the prick'd Lines) and these Intersections are the Points thro' which the Circle must pass, as in the last Example.

This

This Circle here describ'd being the Perspective Appearance of the lesser Circle, we must, to describe the greater Circle, proceed as deliver'd in the preceding Problem.

Perhaps it might been expected that I should have gone through with the Operation of the outer Circle; but in Consideration of the Confusion that would have been caus'd by the many Lines, I therefore thought it much the more instructive, to represent them in two separate *Figures.*

PROBLEM XXI.

Two concentrick (or parallel) Octagons being given, to find their Perspective Appearance in a direct and oblique View, having the Points of Sight and Distance given.

PRACTICE.

1. Let the given Octagons be *y s* H I G F D C, *y s* and N O S *x* Y X U P. *Fig 72.*

2. Continue the Sides H I G F and D C both ways, until they intersect one another in the Points A B E K; also continue the Sides of the lesser Octagon N O, S *x*, Y X, and U P both ways, until they meet in the Points L M W Z, and draw the Diagonals B E and A K.

3. Let A B represent the ground Line, and let the Points 3 2 4 be the given Points of Sight and Distance, and draw the Radials 2 A, 2 B, and Diagonals 3 A, 4 B; and complete the Perspective Square P O A B.

4. Draw the right Line P S, and it will cut the Diagonals in Q R. Then from the Points Q R draw the prick'd Lines R *u*, Q *n*, parallel to A E and B K, up to the ground Line A B. In like manner continue on the Side U P to *r*, and *x* S to *t*, and laying a Ruler from the Point of Sight 2, to the Points *r n u t* in the ground Line A B, draw the Radials *r* 1, *n* 2, *u* 3, and *t* 4, cutting the Diagonals A P and O B in the Points *d e*, 6 *f*, 7 5, and 4 1.

5. Thro' the Points *d e*, draw the Diametrical *d e*, cutting the Radial *n* 2, *u* 3, in the Points *g m*; also thro' the Points 6 *f* draw the Diametrical *h e* cutting the Radial *r* 1, *t* 4, in the Points *e* and *h*. Likewise, thro' the Points 5, 7; draw the Diametrical *h i*, cutting the Radials *r* 1 and *t* 4 in the Points *h i*, and thro' the Points 4 1 draw the Diametrical 4 1, cutting the Radials *n* 2 and *u* 3 in 3 2.

Lastly, To the Points 3 2, *i h m g e h*, draw right Lines, and they will complete the Perspective Octagon 32, *i h m g e h*, which represents the lesser Octagon N O S *x* Y X U P.

Then

Then, to complete the great Octagon without the lesser, proceed as deliver'd in Problem XVII.

As I have now exhibited and made familiar, by great Variety of Examples, all the useful Rules of superficial Perspective, by which the young Learner may readily represent any kind of *Figure* whatsoever, I shall now conclude this Section with four Examples for Practice, *viz.* two Plans of Gardens, and two Plans of Buildings, which will be delightful in their Operations, and give a true Taste of all such Works.

I have omitted to describe the manner of their Operation, with respect to their being perform'd by the same Rules as before delivered, as is apparent by the several prick'd Lines transferr'd from the Geometrical Plan to the Perspective Plan, which determine the Termination of every respective Line and Angle.

It is to be observ'd in all Works of this kind, that the farther you place the Points of Sight and Distance from the ground Line, the better your Work will appear, and especially the Points of Distance,

But, in this Example, I have plac'd the Points of Distance very near to the Point of Sight, (and indeed much too near) that the young Student might the better see the Effect, and thereby, in other Works, know the better how to place them to his own Desire.

Now observe, that this Method here laid down for this direct View is to be understood and practis'd in all oblique Views, as has been already very fully declar'd in the preceding Problems; therefore, having Regard but to the manner of placing your given Plans in Geometrical Squares, when they are not square of themselves, you may with Pleasure, by their Diagonals, represent with great Accuracy all the several Parts thereof, according to the true Rules of Perspective, as requir'd.

SECTION IV.

Of solid Perspective.

Of the manner of representing solid Bodies, as they appear to the Eye, in any View and Distance assign'd.

PRELIMINARIES.

1. TO represent erect solid Bodies, is no more than to represent an infinite Number of Superfices, or rather Plans thereof laid one

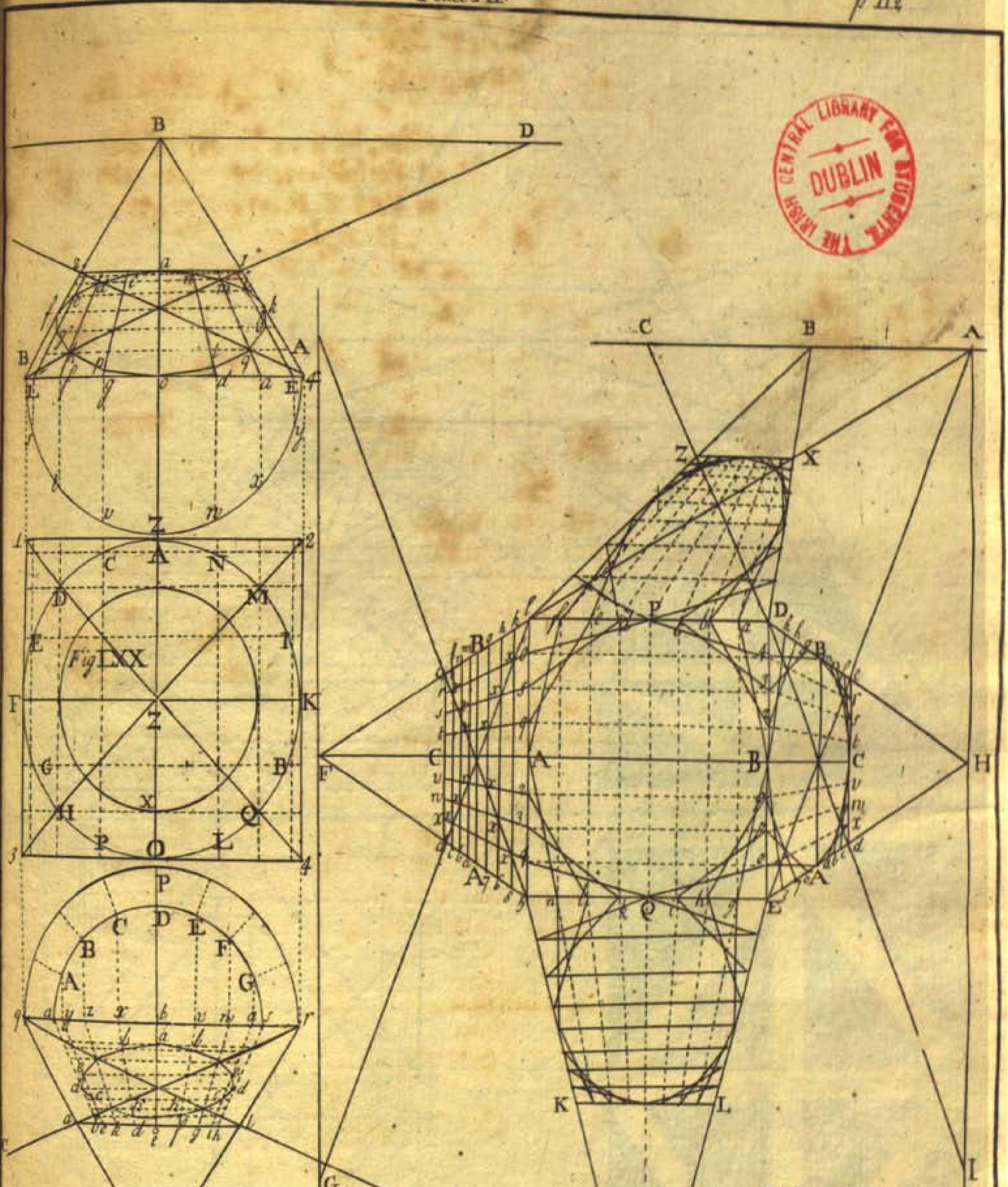


Fig LXX.

Fig LXXI

Fig LXXII.

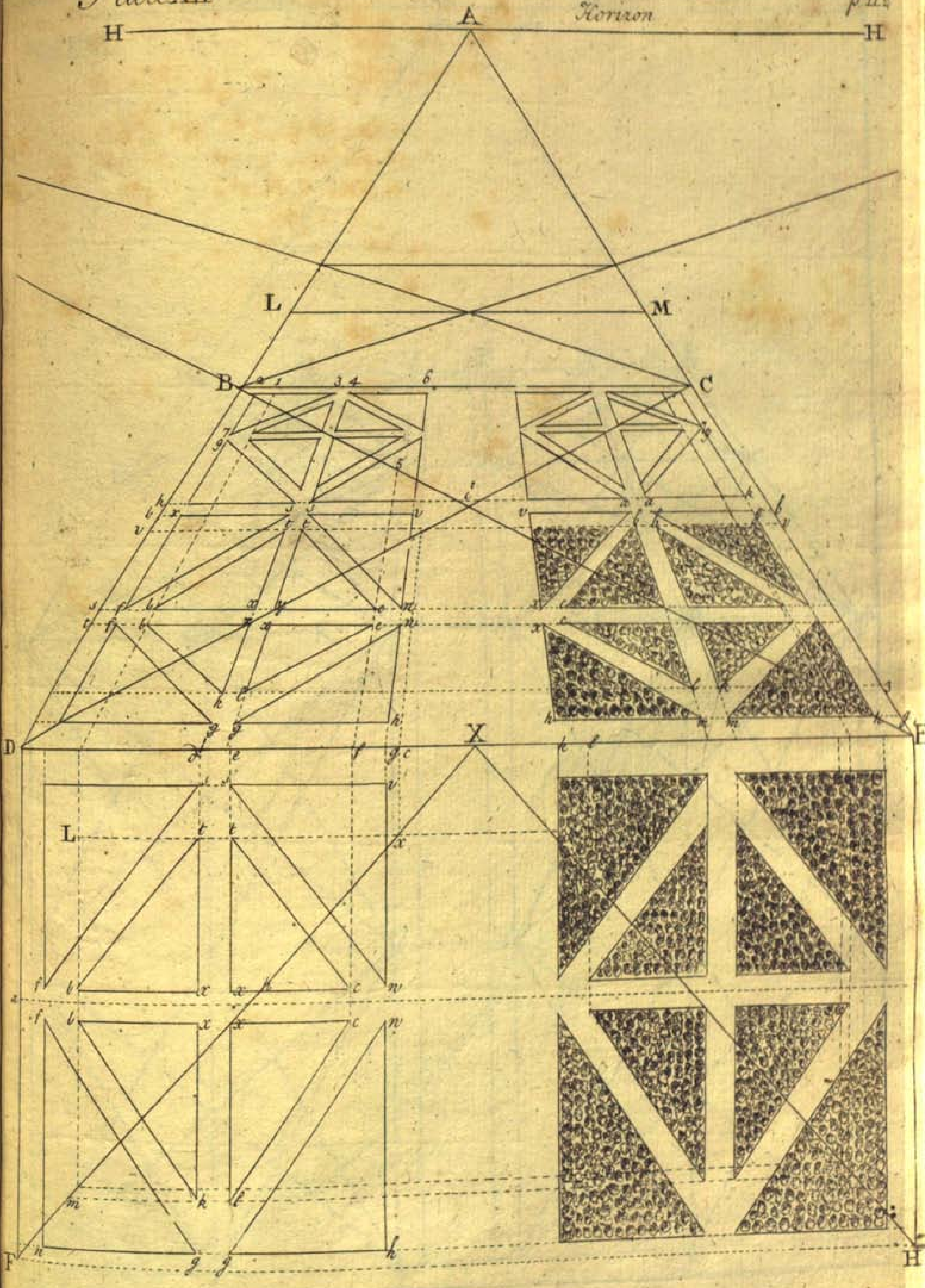

Horizon
A
H
H
L
M
B
C
D
X
E
F
H
L

Plate XIII

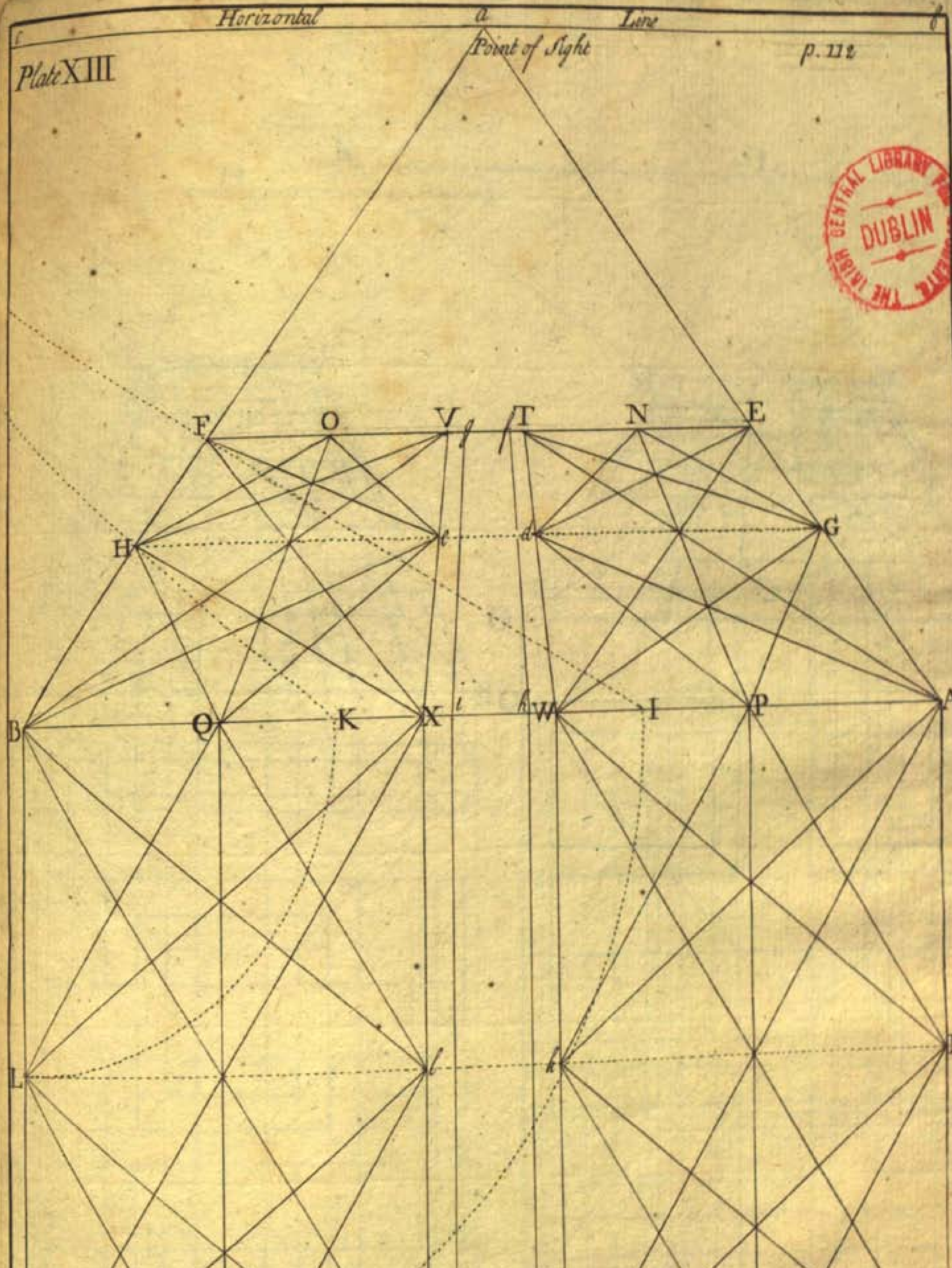
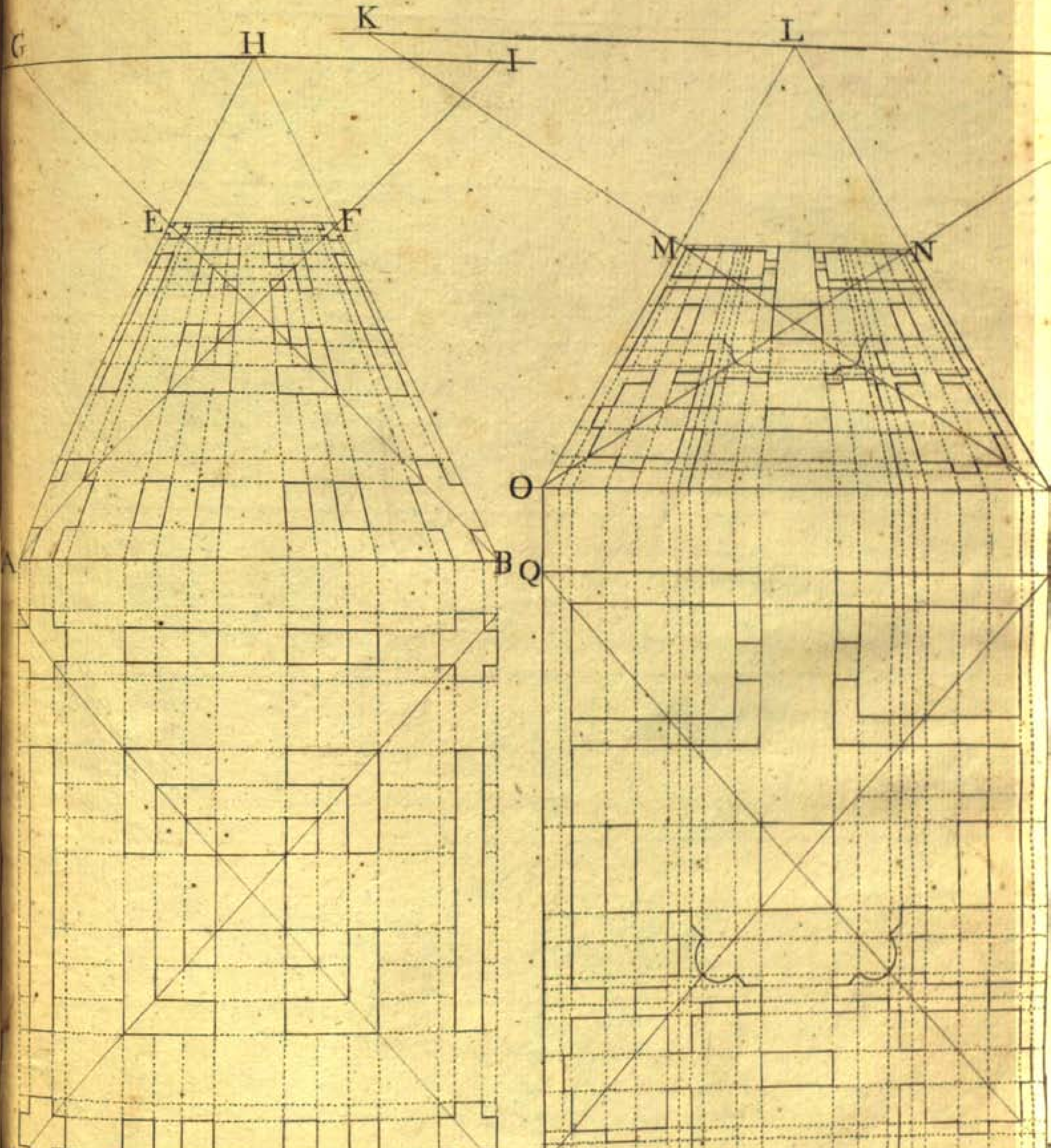

Horizontal
a
Line
Point of Sight
F
O
V
T
N
E
H
G
B
Q
K
X
W
I
P
A
L
M
D
S
Z
Y
R
C

Plate XIV



the use of the Sliding Rule is Described at page 59

Plate XV

one on another. Thus a Cube is no more than an infinite Number of Geometrical square Superfices laid one on the other. (But by the Way, altho' our Geometricans do not allow a Superfices to have Thickness or Depth, yet here 'tis necessary to consider every Plan thus said to be laid one on another, to have a small Thickness, as of Paper, *&c.*) So likewise a Cylinder, or Column, is rais'd by an infinite Number of Circles laid as aforesaid.

2. Before any Solid can be rais'd, the Plan thereof must be given and represented perspectively, as here taught; and then Lines being drawn perpendicular to the Horizon from the Extreams thereof, and terminated with their respective Heights, will represent the Solid requir'd. Thus to represent a Cylinder, or Column, is no more than first to put its Base or Plan in Perspective, and from its Extreams raise Perpendiculars, terminated with their respective Heights, as aforesaid.

3. The finishing of the Heads, or upper Parts of Cubes, Cylinders, Columns, *&c.* is no more than to represent the Form of its Base at the given Height requir'd, be it either below, level with, or above the Eye. But as Columns are of less Diameters at their Tops than at their Bases, therefore to complete the upper Part of a Column, you must perspectively describe a Circle of the same Diameter at the given Height, as is equal to the Diminution of the Colûmn given. These Preliminaries being understood, we may proceed to the Practice following, which is vastly easy, and will enable you to represent the true Perspective Appearance of any Structure with great Exactness and Delight.

PROBLEM I.

A Cube with its Situation, Points of Sight and Distance being given, to represent its Perspective Appearance in a direct and oblique View.

PRACTICE.

First, For the direct View. Plate 15. Fig. 3.

1. Let the Square *t w*, *u x* be the Geometrical Plan of the given Cube *a r b s t u*, and *a b c* the given Points of Sight and Distance.

2. Draw the Diagonals *a t c u*, and the Radials *b t b u*, and complete the Perspective Square *p q t n*.

3. Since the hither Side or Front of every Object that stands directly to the Eye must be made Geometrically true by a Scale of equal Parts, therefore on the ground Line *t u* raise the Geometrical Square *b s t u*, and from its Angles *b s* draw up the Radials *b b*, *b s*.

4. From the Angles *p q* draw up the Lines *p a*, *q r*, intersecting the Radials *b b*, *b s*, in *a* and *r*; then drawing the right Line *a r*, the

Cube is completed as required. For the Perſpective Square *a r b s* is the upper Face thereof, the ſmall Geometrical Square *a r p q*, the Side oppoſite to the Front *b s t u*, and the Perſpective Squares *a b p t*, and *r s q t* are the two oppoſite Sides.

Now ſeeing that the Squares laſt mention'd fall within the Limits of the Front *b s t u*, and upper Face *a r b s*, it is therefore that they are eclips'd by them and can't be ſeen. Hence 'tis evident that when the Eye is plac'd above the Cube, and in a centeral Poſition to one Side direct thereof, it can ſee no more than two Sides at the moſt; and when the Eye is ſo depreſſed, or the Cube rais'd, that the upper, or under Surface thereof lies in the Horizon, as the Cube 6 7 8 9 *a b c*; then there can be but that one Face of the Cube ſeen, which cuts the direct Radial at right Angles, and again the ſame happens in every Degree of Height contain'd in a Cube. For until the Eye is rais'd or depreſs'd above or below the upper or under Surfaces of a Cube, there can be but one Side ſeen; but the Moment that the Eye aſcends or deſcends above or below the upper or under Sides thereof, the top or upper Surface, or bottom or lower Surface inſtantly appears.

And what is here ſaid with reſpect te the Appearances of the upper and lower Surfaces, the like is to be underſtood of the Sides of a Cube. For when the Eye is plac'd, as before, in a direct Poſition to any Part of the Cube, neither of the Sides will appear; but the Moment that the Eye is remov'd unto the Right or the Left, beyond the Limits of the front Lines, the one or other of the two Sides inſtantly appear, and the Cube is then ſaid to be ſeen in an oblique View, as Plate 15, *Fig.* 2 and 3. And as there's two Varieties of direct Views, the one having a Side directing oppoſite to the Eye, as Plate 15, *Fig.* 3, and the other an Angle in the ſame Poſition, as *Fig.* 5; ſo there's alſo the ſame Variety of oblique Views. For, firſt, they may be ſeen obliquely with a Side in front, as *Fig.* 2 and 4, or they may be ſeen with an Angle in front, as *Fig*, 6 and 7; and when Cubes are thus view'd in either of the preceding Poſitions, the Eye is capable of ſeeing three Faces thereof, and is the moſt that can be ſeen in any View whatſoever.

2. *For the oblique View.*

Fig. 2. 1. Let *k l m n*, be the Geometrical Square or Plan of the given Cube to be repreſented, and let *a b c* be the given Points of Sight and Diſtance.

2. Draw

2. Draw the Radials *b k*, *b l*, and Diagonals *k c*, and complete the Perſpective Square *k h i l*.

3. On *k l*, complete the upright or front of the Cube *d g k l*, and from the Angles *d g* draw the Radials *b d*, *b g*.

4. Draw up the Lines *h e*, and *i f*, from the Angles *h i*, paral el to *d k*, and they'll cut the Radials *b d*, *b g*, in the Points *e* and *f*. Then drawing *e f* parallel to *d g*, you complete the Perſpective A - pearance requir'd, wherein is repreſented the Top *e f d g*, and Sides *d e h k*, and *g f l i*.

The Cube *Fig.* 4 is the ſame Cube view'd on the other Side, with all its Requiſites thereto, as *Fig.* 3.

I ſhall now proceed to ſhew the Perſpective Appearance of this Cube, when one of its Angles are plac'd in front, as well in an oblique, as a direct View.

1. *For the direct View.* Plate 15. Fig. 5.

1. Let the Square *m i n p*, be the Geometrical Plan, and the Points *x a z*, the Points of Sight and Diſtance given, and let *k l* be the ground Line.

2. Complete the Perſpective Square *f h i g*, and draw the Radials *a f*, *a g*, and *a i*.

3. Make *i e*, equal to the given Height of the Cube, (that is to any one Side, as *m i*, *a r m p*, &c.) and draw the Diagonals *e z* and *e x*.

4. From the Points *f* and *g*, draw up the Lines *f c* and *g d*, parallel to *a i*, which will cut the Diagonals *e z*, *e x*, in *c* and *d*, and then from the Points *c* and *d*, draw the Diagonals *c x* and *d z*, which will interſect *a i* in *b*.

Laſtly, Draw *c b*, *b d*, and they will complete the Perſpective Appearance required, wherein is repreſented the Top *c b e d*, and Sides *c e f i*, *e d i g*.

2. *For the oblique View.* Fig. 6.

1. Let the Square *m e o n* be the Geometrical Plan as before, and let the Points of Sight and Diſtance be the ſame.

2. Draw the Radials *a l*, *a k*, and Diagonals *z k*, *x l*, and complete the Perſpective Square *i d k l*.

3. Upon the Angle *e*, raiſe the Perpendicular *e c*, which make equal to the given Height of the Cube, (as you before did *e i*,) and from the Point *c*, draw the Lines *c a* and *c x*.

4. From the Points *d* and *i*, draw up the Lines *d b* and *i g*, parallel to *c e*, until they meet the Lines *c z* and *c a* in the Points *b* and *g*.

5. Draw the Lines *b a* and *g z*, intersecting each other in *f*, and they complete the Cube requir'd, wherein is represented the Top *f b c g*, and Sides *b c d e*, and *c g e i*.

The Cube *Fig.* 7 is the same Cube view'd on the other Side, with all its Requisites thereto belonging.

These and such like Views here represented, are the several Varieties in which a Cube may be seen, when 'tis standing upon one of its Sides for the Base, (as all Buildings do.) But since it may happen, that in some Works it may be requir'd to represent them lying upon an Angle for their Base, as Plate 15, *Fig.* 8. I will shew the Method of delineating them in that Position as follows.

PROBLEM II.

To represent a Cube, lying upon one of its Angles for its Base in a direct View, having the Points of Sight and Distance given.

PRACTICE.

Plate 15 *Fig* 8.

1. Let *x x* be the ground Line, with the Geometrical Square *g e i h*, placed thereon, with its Diagonal *e h* perpendicular thereto, and let this Square represent the hither End or Point of a Cube, lying upon the Angle *h*.

2. Let the Point of Sight be *a*, and Point of Distance *b*, and let the Radials *a g*, *a e*, and *a i* be drawn.

3. Draw *d f* thro' the Angle *e*, parallel to the ground Line *x x*, and laying a Ruler from the Point *d* to the Point of Sight *b*, it will intersect the Radial *a h* in *c*.

4. Draw the Lines *d c*, and *c f*, and they will complete the Perspective Appearance required.

The Cube *l k n m p o*, is an oblique View of the same Cube, standing on the ground Line *z z*, which you see hath its Angles *l k* determin'd by the continuation of the Lines *d f*, and *c k*, until they meet the Radials *a m* and *a n* in the Points *l* and *k*.

PROBLEM III.

To represent a long Cube or Parallelopipedon, standing perpendicular to the Horizon, having the Points of Sight and Distance given.

PRACTICE.

☞ Under this Figure all kinds of Pillasters are represented, therefore observe, that what is here said of a Parallelopipedon, or long Cube, the same is to be understood of a Pillaster, whose Diameters at top and bottom are equal.

The several different Positions in which an upright Parallelopipedon may be plac'd, are exactly the same as the preceding of the Cube. As

first,

first, they may stand in a direct View with one Side to the Eye, as Plate 17, *Fig.* 3. or with an Angle next the Eye, as *Fig.* 2.

And secondly, they may stand in an oblique View, with a Side or an Angle in the aforesaid manner, as Plate 16. *Fig*, 8.

To delineate the Parallelopipedon in these several Views, I will illustrate the same by Examples.

EXAMPLE I.

Let it be required to represent in a direct View a Parallelopipedon, whose Altitude is equal to the given Line D C, and Side of it; Base to the Line E F, Plate 17. *Fig.* 3.

PRACTICE.

1. Let *i k* represent the ground Line A B C, the Horizon and the Points A B C, the Points of Sight and Distance.

2. Make *n h*, *n g*, each equal to half E F, and complete the Parallelopain *a b g h*, making the Height *a g*, equal to the given Height D C.

3. Draw the Radials, as well B *a* B *b*, up to the upper Part *a b*, as B *g* and B *h*, down to the Base or ground Line *g h*, as also the Diagonals A *b*, A *h*, and C *g*, C *a*, and complete the two Perspective Squares *a b c d*, and *e f g h*.

Then from the Angles *e f*, *c d*, draw the Lines *c e*, *d f*, and they complete the Perspective View required.

Now 'tis observable, that as the Side *c d e f*, (which is the opposite Side to *a b g h*) is comprised within the Limits of *a b g h*, it is eclips'd thereby, and can't be seen; so likewise are the two opposite Sides *a c g e*, and *b d f h*. Therefore, when a Parallelopipedon is thus situated there can be but one Side seen thereof, as was before declared in the like Position of the Cube.

EXAMPLE II.

Let it be required to represent a Parallelopipedon, whose Altitude and Side of the Base is given, with an Angle thereof plac'd in a direct View, as Plate 17. *Fig.* 2.

1. Let *m n o*, represent the ground Line, C D E the Horizon, and the Points of Sight and Distance at C D E. Complete the Perspective Square *f h n p*, with its Sides each equal to the given Side *z*. Then on the Points *p k f* raise the Perpendiculars *f b*, *h* B, and *p f*. and make each of them equal to the given Height of the Parallelopipedon X X.

2. Draw the upper Radials *b* D, *d* D, and *f* D, as also the Diagonals *a* E, *g* C, &c. and complete the Perspective Square *b* B, *d f*, and

and thus is the Pillaster completed, with an Angle plac'd in a direct View as required.

Plate 17. Fig. 1.

EXAMPLE III.

Let it be required to represent the same Pillaster in an oblique View, with its Angle plac'd as before.

1. Let the Angle opposite to the Eye be plac'd at *o*, and then complete the Perspective Square *h o l m*, with its Sides respectively equal to the given Line Z.

2. On the Points *h* and *m*, raise the two Lines *o b* and *m x*, perpendicular to the ground Line *r n*, and equal to their given Height; then at their upper Parts *c d*, draw the Radials A D, *a* D, &c. to the Point of Sight D, and complete the Perspective Square *b d e c* in every Respect as you did the Perspective Square *h l o m*.

3. Draw the Lines *d h*, *o b*, *e l*, and *c m*, and they complete the Perspective Appearance required.

EXAMPLE IV.

Let it be required to represent the Pillaster *a b d i m l*. Plate 16. *Fig.* 8. in an oblique View, with one Side thereof, as *i m* in the ground Line *i c*.

1. Let the Geometrical Square *i m n o* be the Base of the Pillaster, the Point A the Point of Sight, and the Line A 8 the Horizon,

2. Complete the Perspective Square *i k l m*, and from the Point *i m* draw the Lines *i a*, *m b*, perpendicular to *i m*, and each equal to the given Height, and draw *a b* the Head of the Pillaster.

3. From the Points *i m*, and *a b*, draw the Radials *i* A, *m* A, and *a* A, *b* A unto the Point of Sight A, and complete the Perspective Squares *i k m l*, and *a b c d*; then drawing the Line *d l* and *c k*, you complete the Pillaster required. Now 'tis observable, that the nearer an Object approaches the direct View, the less Quantity of the Side is seen: For the Side *g h s r* of the Pillaster *p e g h s*, has a lesser Appearance than the Side *b d m l* of the Pillaster *a b d i m l*, &c. altho' of the same Magnitude.

It is to be observ'd, that Objects whose upper Parts are above the Horizon, as the two preceding Pillasters, their upper Surfaces cannot be seen; and when the Top of a Pillaster lies exactly in the Horizon, as the Pillaster 6, 7, 8, 9 *b c*, *Fig.* 8. then the upper Surface appears but as the right Line 6, 7, 8; but when the upper Part of a Pillaster lies under the Horizon, as *w x y z*, then it is visible, and at no other Time.

PRO-

PROBLEM III.

To represent a long Cube or Parallelopipedon lying upon the Ground in any View required.

A Parallelopipedon is represented by continuation of a Cube, first made and plac'd in the given Position, as follows.

EXAMPLE I.

Let it be required to represent a long Cube, or Parallelopipedon, with one of its Sides in an oblique View, and whose Length shall be equal to *d* 6. Plate 15. *Fig.* 4. and Side of its Square to the Height 2. 6.

1. Let the Cube *y z* 1 2, 3 5, 6 be given, let *b* be the Point of Sight, and *a* the Point of Distance.

2. Continue the Side 6, 5 to the given Length *d* 6, as also 1 2, to *b*, and complete the Side of the Parallelopipedon *b* 2 *d* 6.

3. Draw up the Radials *d b*, and *b b*, to the Point of Sight *b*, and then continuing *z y* until it meet the Radial *b b* in *a*, it will complete the upper Face of the Parallelopipedon.

Lastly, From the Point *a*, draw the Line *a c* parallel to *b d*, and it will cut the Radial *b d* in *c*; or otherwise continue on the Line 3 4, and it will cut the Radial *b d* in *c*, then drawing *a c*, the Parallelopipedon is completed as required.

EXAMPLE II.

Let it be required to represent a Parallelopipedon in an oblique View, as the Cube *d e f g*, *k l i*. Plate 15. *Fig.* 2.

1. Let the given Length be *l i*, and Height *k d*, or *g l*; and complete the End *g d l k*.

2. Draw the Radials *b l*, *b k*, *b g*, and *b d*, and make the Radial *l b* equal to the given Length at *i*, and *f m* the Point *i*, draw up the Line *i h*, parallel to *l k*, which will cut the Radial *g k* in *h*.

3. From the Points *i* and *h* draw the Lines *i f*, and *h e*, parallel to *g l*, and lastly draw *f e* parallel to *g d*, and it will complete the Parallelopipedon as required.

Now, from these three Examples it is to be noted, that to represent a Parallelopipedon in any given Position, and then the Sides continuing to the given Length, may complete it with Ease as requir'd.

But since that these Methods have recourse to Parallelopipedons, whose Sides are supposed to be exactly square, which do not always happen in Practice, therefore I shall now proceed to the manner of repre-

representing such Parallelopipedons whose Bases are Parallelograms, instead of Geometrical Squares, as before.

PROBLEM V.

To represent Parallelopipedons, whose Bases are Parallelograms.

Plate 17. Fig. 4. 1. Let the Line 4 *k* 5 be the Horizon, *k* the Point of Sight, and let 3 2 *q p d c* be the ground Line, also let the Parallelogram *c d e f* be the End of the given Parallelopipedon.

2. Let the End of the Parallelopipedon given be plac'd at Pleasure on the ground Line, as *c d*, and then making the Parallelogram *a b c d* equal to the Parallelogram *c d e f*, draw up the Radials *a k*, *b k*, and *d k*.

3. Let the given Length be equal to *d g*; then from *g* draw up the Line *g i*, parallel to *b d*, until it cuts the Radial *b k* in *i*, and so will the Parallelopipedon be completed as required.

Now from this it appears, that after the Base *a b c d* is determin'd, and the Radials drawn, the following Operations are directly the same as before in square Parallelopipedons.

Fig. 5. is a direct View of the same Parallelopipedon, and *Fig.* 6. an oblique View different from that of *Fig.* 4. which are both determin'd by the aforesaid Rule.

I shall now proceed to apply the several Rules of this Section to Practice in the Delineation of the Perspective Elevations of Buildings. First, as they oftentimes appear when fram'd with Timber only, and Lastly, when completely finish'd with Brick, Stone, &c.

CHAP. II.

Of the manner of representing the Perspective Elevations of Buildings in general.

PROBLEM I.

TO represent the Perspective Appearance of a fram'd Cube being seen in a direct and oblique View, as Plate 16. *Fig.* 3 and 4.

PRACTICE.

Fig. 3. 1. For the direct View.

The manner of representing this Figure is the very same of Plate 15. *Fig.* 3, as is discover'd at first View, save the Thickness of the framing only, as will now appear by the Operation.

PRACTICE.

1. Suppose the Line 1 2 to be equal to one Side of the Cube, and *w* 2 to one Side of its Thickness.

2. By the preceding Problems, complete each Side of its Bottom, as so many distinct Parallelopipedons, and from their several Ends raise the four Standards, as four several erect Pillasters standing thereon, of which the two in front have their Heights determin'd by the given Side of the Square, and the other two by their visual Rays, meeting them in *a b* and *c d*.

3. Drawing right Lines from each return of the Heads of the four erect Pillasters, unto the respective return of the others, they will complete the fram'd Cube as requir'd.

2. *For the oblique View.* Fig. 4.

1. Let *m n* be equal to the Side of the given Cube, and *o n* equal to the given Thickness of the framing.

2. At *m o* and *o n* raise the two erect Pillasters *g m*, *r m o*, and *h k*, 3 *o n*, and joining their respective returns by right Lines, you will complete the front of the Cube.

3. From the Extreams of the Parallelopipedon 5 3 *r m n t*, draw right Lines to the Point of Sight B, and where its Plan terminates erect the farther Standards *a b* and *f* 8, and complete them as erect Pillasters, whose Breadth or Thickness are terminated by the radial Lines *p m*, *y z*, *y* 4, 1 2, &c.

4. Their respective returns being join'd by right Lines, completes the whole as requir'd.

PROBLEM II.

A Geometrical Square in Perspective being given, with a Pillaster in each of its Angles of a given Height, to raise the Perspective Elevation in a direct View. Plate 16 Fig 7.

PRACTICE.

1. Let *k* 6 4 5, 7 *x* 8 9, *z* 3 1 2, and *a z c n* be the Plans of the given Pillasters, plac'd as beforesaid, and let *a* 1, or *b n* be equal to the given Height.

2. From the aforesaid Plans of the given Pillasters, raise Lines perpendicular to the Base 1 *n*, and parallel to each other, and complete the two Pillasters in front according to their given Heights.

3. Draw the Radials *c* A, *d* A, *b* A, and *e o s* A, *f* A, *g* A, which will terminate the Height of the two backward Pillasters.

4. Their respective returns being join'd by right Lines, completes the whole as required.

The *Figure* 6 of Plate 16, and *Figure* 2 of Plate 18, are Views of the same, the first in an oblique View to one Side, the other in a direct View to one Angle, which are both rais'd and completed by considering each Part as a single Pillaster, and therefore needs no further Explication.

PROBLEM III.

There is an oblong Plan given in a direct View, with the Plans of two Ranges of Columns or Pillasters, parallel to each other to raise the Perspective Elevations thereof, supporting Semi-circular Arches, as Plate 15, *Fig.* 1.

PRACTICE.

1. Let the given Plan be Plate 18, *Fig.* 1. and the Plans of the Pillasters *n* 6 *o p*, *y* 5 *w x*, 1 2 *i k*, 3 4 *l m*, *&c.* and A the Point of Sight, also let *a n* be the given Height of the two Pillasters in front.

2. From the Points *n p w x*, raise the Perpendiculars *n a*, *p* 6, *w c*, *x d*, making each equal to the given Height *q n*, and draw the Line *a b c d*. This done, bisect the Line *b c* in B, which is the Center of the Arches *a x d*, and *b y c*.

3. *Thirdly*, Draw the Radials *b* A, *c* A, and raising the Perpendiculars *o s*, and *y u*, complete the Pillasters *a b s*, *n o p*; and *u c d*; *y w x*, and then drawing the Diameter *r t*, bisect it in C; the Point C is the Center of the Semi-circle *r z t*, and so is the first Arch completed.

4. From the Points *k* and *l*, raise the two Perpendiculars *k h* and *l e*, intersecting the two Radials in the Points *h* and *e*, thro' which draw the right Line *g h e f*; and raise up the Perpendiculars *i g*, *m f*, as also the Lines 2 *a* and 3 *b*. Plate 15 Fig. 7.

5. Bisect *h e* in D, and *a b* in E, then shall D be the Center of the Arches *g* 1 *f*, and *h* 2 *e*; and E the Center of the Arch *a* 3 *b*, and so is the second Arch completed.

Lastly, Proceed in like manner with all the others, and you'll complete the whole as required.

PROBLEM IV.

The preceding Plan given in an oblique View, as Plate 19, *Fig.* 1. to raise the Perspective Elevation thereof.

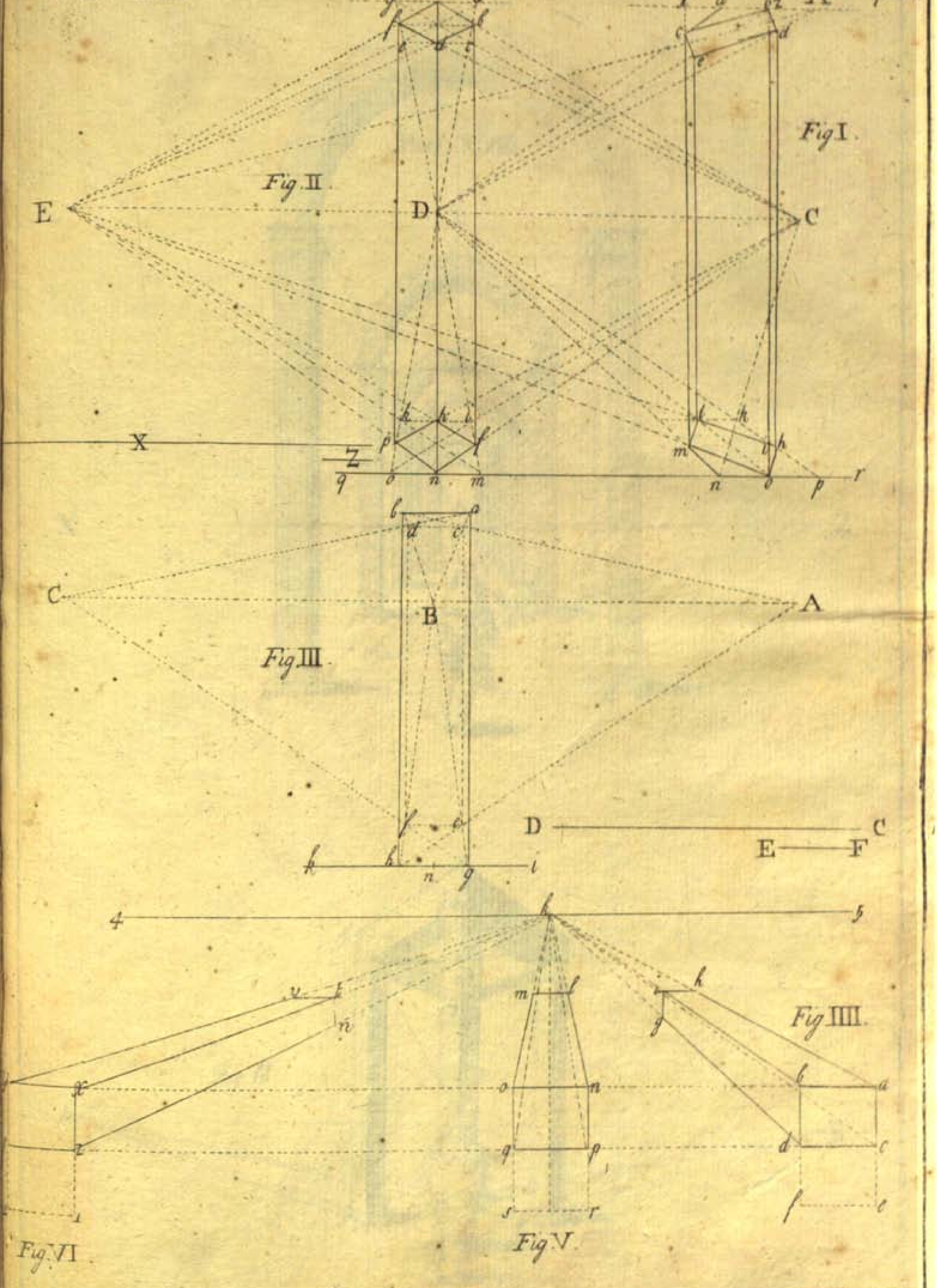
1. Let the Point of Sight be A; A B the Horizon, and E *x b f* the given Plan.

2. From the given Angles *a b c*, and *d e f*, raise the Perpendiculars *a q*, *b* R *c s*, and *d x*, *e y*, *f z*; and make each of them equal

to


Fig. I.
Fig. II.
Fig. III
Fig. IV.
Fig. V.
Fig. VI.
Fig. VII.
Fig. VIII.

Plate XVII



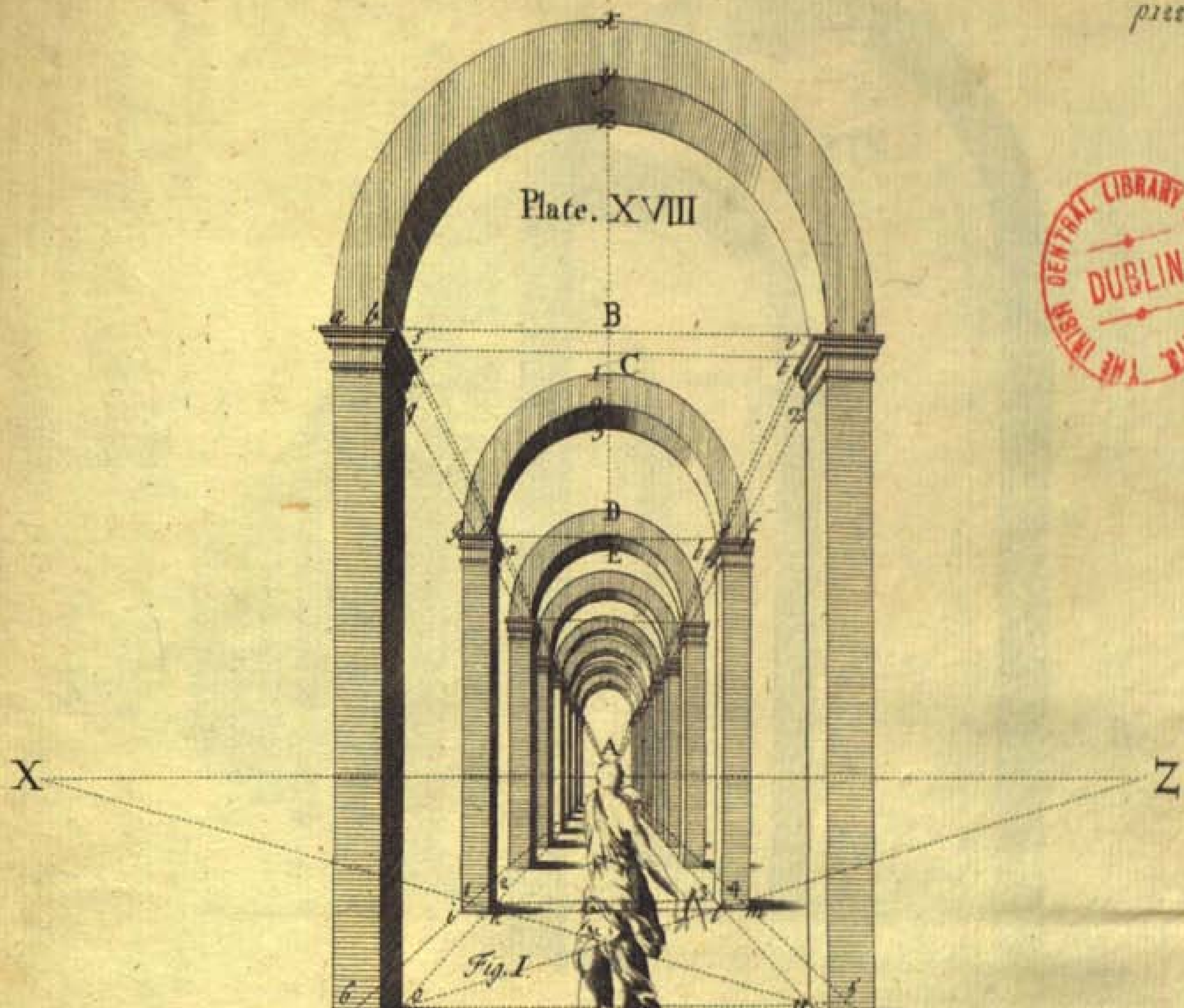


Plate. XVIII
X
Z
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D
E
Fig. I.

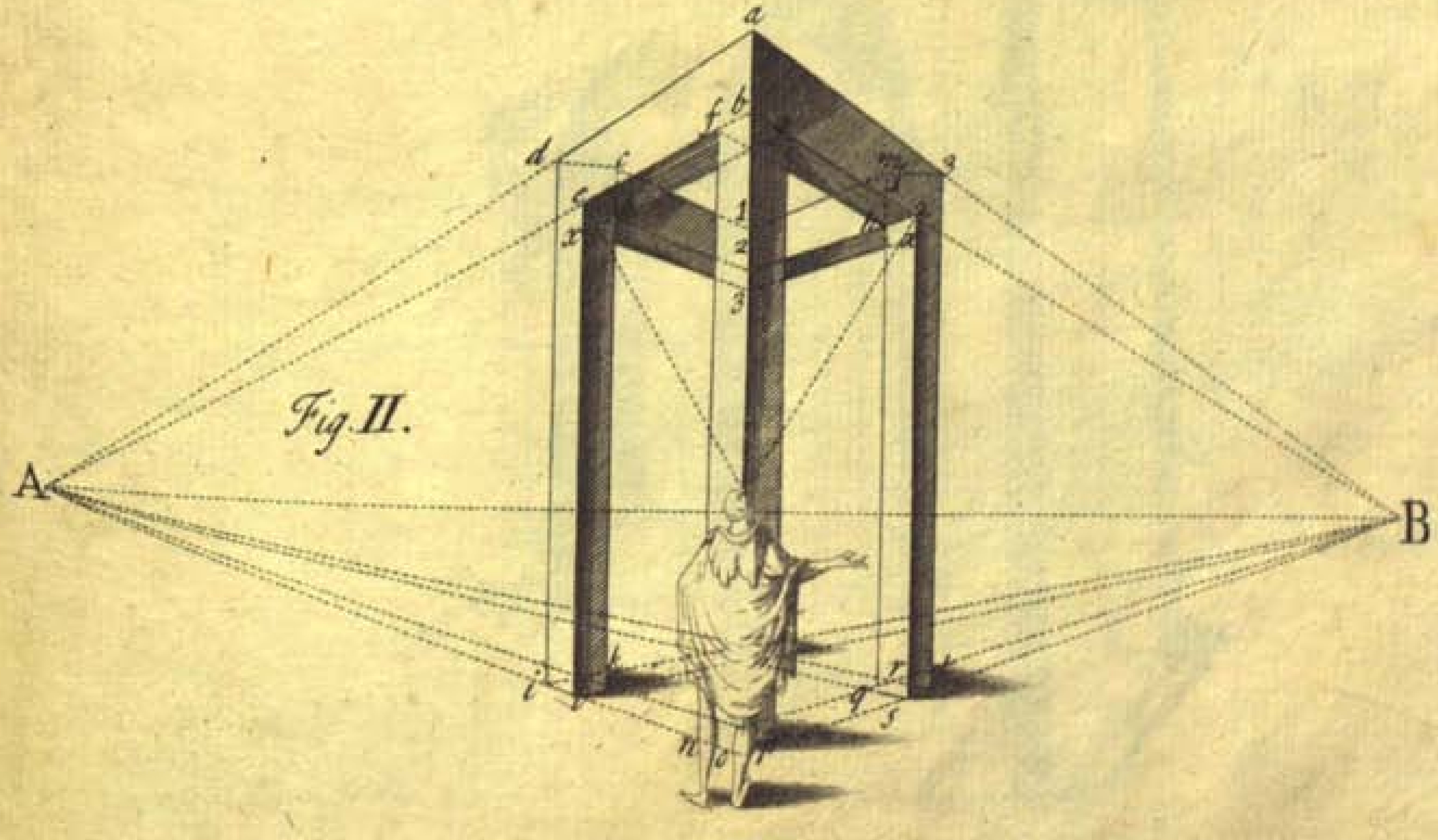


Fig. II.
A
B


Plate. XIX
Fig. I
A
B
C
D
E
R
Fig. II
A
B
C
E
F

the given Height of the Pillasters, as *b p*, or *f z*, and draw the head Line *s z*, which bisect in *h*, the Center of the Arches R *t z*, and *s u y*.

3. From the Points R and *y* draw right Lines to the Point of Sight A, then will these Radials *y* A, and R A, determine all the Heights of the other Pillasters at the Points 1 2, and *n c*.

4. From the Points *x* and *q*, where the Perpendiculars *d x* and *a q*, meet the radial Lines *y* A, and A, draw the Line *q x*, which bisect in *n*, the Center of the Arch *q r*.

5. From the hindermost Angle E of the Pillaster *a* E *b c*, raise the Perpendicular E E, meeting the Line *x* E in E, then bisect E *x* in *o*, and 'tis the Center of the Arch *x w* E, which completes the first Arch required.

This done, from the Angles *g h i*, and *k l m*, raise up Perpendiculars as before, and from the Points 1 and *n*, where the Perpendiculars *k* 1 and *h n* meet the Radial *y* A, draw the head Line 1 *n*, and then proceed to find the Centers of the several Arches, as in the preceding, and you'll complete the second, third, &c. Arches in like manner as required.

PROBLEM V.

The preceding Plan given in an oblique View, with an Angle plac'd in front, as Plate 19. *Fig.* 2. to raise the Perspective Elevation thereof.

1. Let A be the Point of Sight, B the Point of Distance, and A B the Horizon; also let the Plans of the given Columns be *g h i k*, *l m n o p r s*, *t u w x*, &c.

2. From the front Angle *i* raise the Perpendicular *i a*, and make *i a* equal to the given Height of the Pillaster, and from *a* to A the Point of Sight, and B the Point of Distance, draw the Lines *a* B, *a* A.

3. From the Angle *n*, raise the Perpendicular *n b*, meeting the Radial *a* A in *b*, and from the Point *b*, draw the Line *b* B.

4. From the Angles *g k l o*, raise the Perpendiculars *g c*, *k d*, *l e* and *o f*, meeting the Line B *a* in *c a*, *b* A in *e*, and *a* B in *d* and *f*, and so will the first two Pillasters be completed.

N. B. The Heights of the Arches, that is, the Heights of the Points C and B, above the Point *a*, are the Geometrical Heights of the same Arches taken from the front Arches, as the Height *t u*, *Fig.* 1. which being seen in this oblique View, doth appear no higher than E as in the Figure, altho' 'tis seen under the same Angle with B.

The same Operations being perform'd at the other Arches, you will complete the whole as required. But lest this Problem may put the young Student to a Stand in the Arches, I will make the Nature thereof more plainer by the following Problem.

PROBLEM VI.

To describe Semi-circle Arches in side Views, either direct or oblique.

Plate 20. First in a direct View.

Let 2 1 4 3, *Fig.* 1. be the Perspective Plan given; and let *z* be the Point of Sight, and let it be required to delineate the Arch *k l m n d o p q* B, standing over the Side 2, 4; which will be equal in Appearance to the front Arch A F H K B.

1. From the Angles 3, 4, raise the perpendicular Lines 3 A, and 4 B, making their Heights equal to the Height assign'd, as 3 A or B 4, and draw A B, bisecting it in X, the Center whereon with the Radius X B, or X A describe the Semi-circle A H B.

2. Draw C D parallel to A B, and continue 3 A to C, and 4 B to D.

3. From the Angles 1 and 2, raise the Perpendiculars 1 *z* and 2 *k*, which continue up until they meet the Radials A *z* and B *k* in *z* and *k*, and drawing the Line *z k*, bisect it in *m* the Center, and thereon describe the Semi-circle *z n k*.

4. Divide the Semi-circle A H B, into any Number of equal Parts, as 8, 10, 12, 14, 16, *&c.* But in this Example I have divided it into eight, at the Points E F G H I K L, from which Points draw right Lines towards the Point of Sight Z, until they meet the Semi-circle *z n k* in the Points *o o o*, &c.

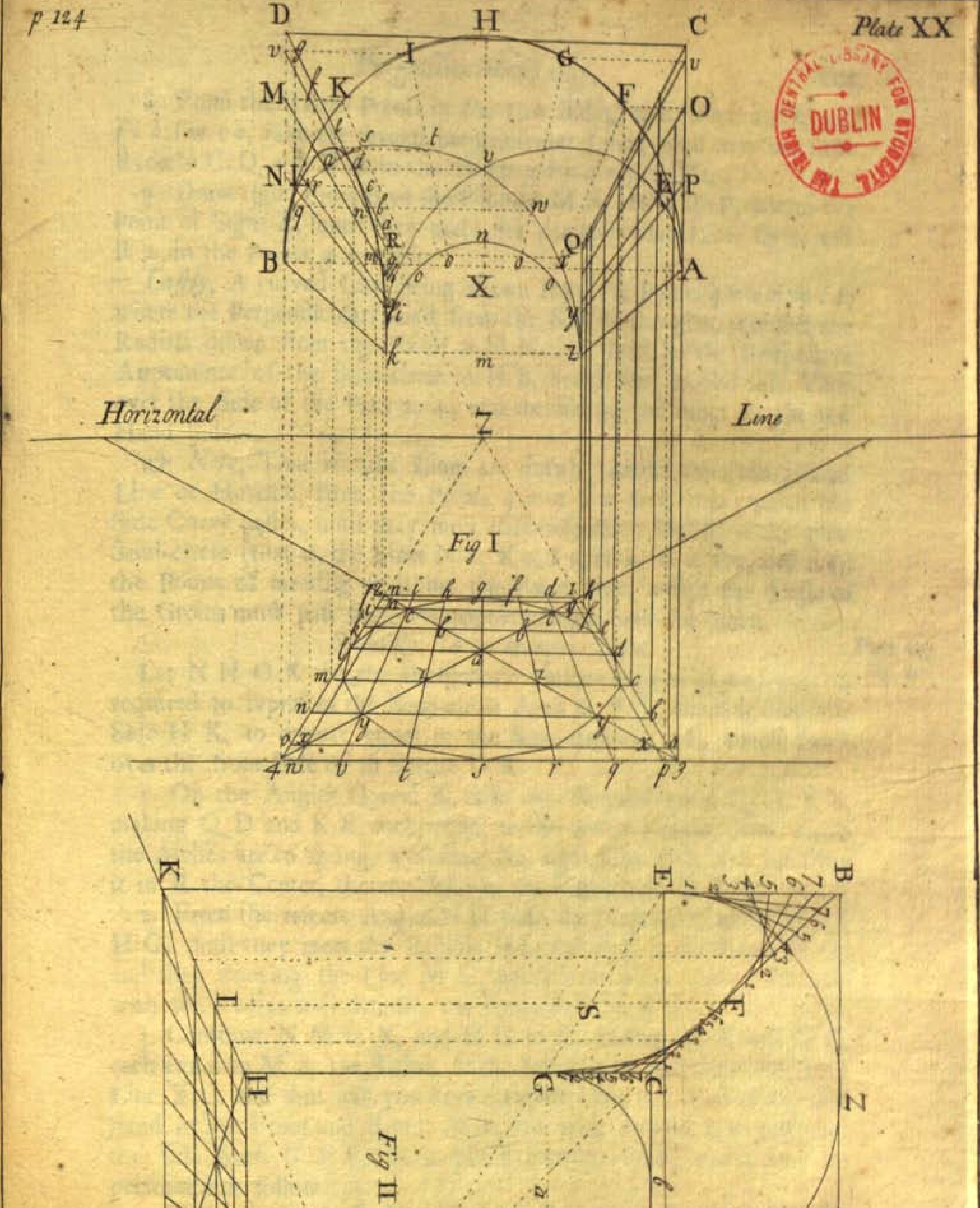
5. From the Points G and I draw right Lines both ways, as well to the right as left Hand, as G *u* and I *u*. Also from the Points F and K, draw the Lines F O and K M, and lastly from the Points E and L, the Lines E P and L N.

6. Let fall perpendicular Lines from the Points E F G I K L upon the ground Line 3 4, and they will cut it in the Points *p q r s t u w*, from which, Lines drawn to the Point of Sight *z*, will cut the Diagonals in *x x*, *y y*, *z z*, *a*, *b b*, *c c*, *g n*.

7. Thro' these several respective Points draw right Lines, as *x x*, cutting the Side 1, 3, in *a*; and 2, 4, in *o*; also the Line *y y*, cutting the Sides in *b* and *n*; *z z*, cutting the Sides in *c* and *m*; the Diameter *a* cutting the Sides in *d* and *l*; *b b*, in *e k*; *c c* in *f i*; and *g n*, in *f* and *h*.

8. From

Plate XX



8. From the several Points in the two Sides, *viz a b c d e h*, and *f i k l m n o*, raise the several perpendicular Lines, until they meet the Radials C Q and D R in the Points *a b c d e f g*, &c.

9. Draw right Lines from the Points *u* M N, and *u* O P, *u n* to the Point of Sight Z, until they meet the perpendicular Lines Q 1, and R 2, in the Points *g h o*, &c.

Lastly, A curved Line being drawn thro' the Points *q r a n m l k*, where the Perpendiculars rais'd from the Side of the Plan, intersect the Radials drawn from the Points *u* M N, *&c.* shall be the Perspective Appearance of the Semi-circle A H B, being seen in the side View over the Side of the Plan 2, 4; and the like of the other on the left Hand Side.

☞ *Note*, That if right Lines are drawn parallel unto the ground Line or Horizon, from the Points *q r a d n m l*, thro' which the Side Curve passes, until they meet their respective Radials of the great Semi-circle (that is, the Lines N *o*, K *o*, I *o*, H *n*, G *o*, F *o*, and E *o*,) the Points of meeting shall be the Points thro' which the Angle of the Groins must pass from one opposite Angle unto the other.

Secondly, *In an oblique View*. Plate 20. Fig. 2.

Let N H O K be the Perspective oblique Square given, and 'tis required to represent the Semi-circle Arch G F E, standing over the Side H K, to appear equal to the Semi-circle D Z E, which stands over the front Side of th Square O K.

1. On the Angles Q and K, raise the Perpendiculars Q A, K B, making Q D and K E, each equal to the given Height, from which the Arches are to spring, and draw the right Line D E, and bisecting it in R the Center, thereon describe the Semi-circle D Z E.

2. From the remote Angles N H raise the Perpendiculars N M and H G, until they meet the Radials D L and E L in the Points M G; and then drawing the Line M G, bisect it in *a* the Center, whereon with the Radius *a* G describe the Semi-circle M *b* G.

3. Continue N M to X, and H G to C, making M X and G C, each equal to M *a*, the Radius of the Semi-circle, and draw the Head Line X C, and thus will you have completed the two Semi-circles that stand in the Front and Rear. Now, the next Business is to complete the Side Arch G F E that is plac'd between them, which may be perform'd as follow.

1. From the Angle B, draw the Line B C to the Point of Sight L, terminating at the Angle C, and then you will have the Perspective Parallelo-

Parallelogram C B G E, wherein you are to delineate the Semi-circle G F E, standing on the Lines G H and E K.

2. From the Point I in the Side H K of the Plan, raise the Perpendicular I F, cutting the Radial L B in F.

3. Divide C F, F B, B E, and R G, each into any Number of equal Parts, and then proceeding as directed in Plate 11, *Fig.* 71. you will complete the rampant Curve G F E as required.

☞ *Note*, When these Curves are requir'd to a very great Accuracy, you must divide the Side of the Plan H K into as many equal Parts, by Diametricals drawn thro' the Intersections of the Radials produc'd from the equal Divisions first made on the ground Line Q K and Diagonals) as you find necessary, from which you must draw up perpendicular Lines parallel to B K until they meet and divide the radial Line R B in the same Proportion. For tho' the Curve made by the equal Divisions in G C, C F, F B, and B E, will do for most Occasions, yet it is not the true Curve which may be produced from the Divisions aforesaid, and will be something different from the Figure here laid down, altho' but very little in small Works.

PROBLEM VII.

The Geometrical Plan of a Building being given to raise any Perspective Elevation thereon, from a Geometrical Elevation given in a direct View.

1. Let the Geometrical Plan be U W, T X, P S, Q R. *Fig.* 3.

2. Let X Y represent the ground Line, A B the Horizon, A and B the Points of Distance, and D the Point of Sight.

3. Draw the Radials D X, D Y, and the Diagonals X B, Y A, and complete the Perspective Plan *u w t x p s q r*.

4. Draw the Line H H. *Fig.* 1. for the ground Line of your Perspective Elevation; as also A C for the Horizon thereof, at the same Distance from the ground Line H H, as A B of *Fig.* 2. is from the ground Line X Y, and let B represent the Point of Sight, and A C the Points of Distance.

5. Make K G, *Fig.* 1. equal to Z Y, *Fig.* 2. and complete the hither half of the Perspective Plan, that is, you must represent in *Fig.* 1. first the Line *m n*, equal to *q r* in *Fig.* 2. and secondly, the Line *l o*, equal to 6 *s*, from which Angles you are to raise the Elevation as follows.

First, From the Angles *p q r s*. *Fig.* 2. raise the perpendicular Lines *p l b*, *q m i*, *r n k*, and *s o g*; cutting the Line *m n*. *Fig.* 1. in the Points *m* and *n*, and the Line *l o* in the Points *l o*.

Secondly,

Secondly, Upon the Line *m n*, raise any Front at Pleasure Geometrically, according to the usual Method, by the same Scale that the Geometrical Plan U W, T X, P S, Q R, was laid down, by placing therein such Doors, Windows, *&c.* as your Fancy leads, as the Geometrical Upright *i k m n*, whose Height from the ground Line *n* to the Cornish *k* is 30 Feet.

Thirdly, From the Angles *l* and *o*, *Fig.* 1. raise the perpendicular Lines *l b*, *o g*, making each of them equal to 30 Feet, the before given Height of the Cornish, and draw the Eves Line *b g*.

But here Note, That as the Line *l o* stands back from the Front *m n*, it doth thereby appear less, and lower as being seen under lesser Angles, and therefore the Height *b l* and *o g* of 30 Feet each must not be set off with the same Scale as the Front *m i*, or *k n* were, but by a new Scale of Feet proportionable thereunto, which is thus found.

To proportion a Scale of equal Parts to any Part of a Perspective Plan and Elevation required.

RULE.

Set the Scale by which the Geometrical Plan was laid down, upon the ground Line of the Perspective Plan, and Radials being drawn from the respective Divisions thereof to the Point of Sight, will cut and divide every Part of the Perspective Plan proportionably. Suppose Z G in the ground Line *Fig.* 1. be equal to ten Feet of the Scale, by which the Geometrical Plan was laid down. Then I say, that if the Radials G B, Z B, are drawn, they will cut the Base Line *l o* in *x o*, then will *x o* have the same Proportion to the whole Line *l o*; as Z G, has the whole Line G H, and therefore *x o* is a Scale of ten Feet to the Line *l o*, by which the two Heights of 30 Feet each, *viz.* *l b*, and *o g* may be proportion'd as required.

Having thus by this new Scale determin'd the Height of the Cornish, you must now proceed to put in all such Windows, *&c.* thereby, as if it was a Geometrical Elevation, and above the Cornish place the Height of the Roof, Geometrically giving it its natural Hips, and draw the ridge Line *a f*.

Fourthly, Lay a Ruler from the Point of Sight B, to the Points *i* and *k*, and draw up the Eve Lines *i d* and *k c*, until they meet the Eve Line *b g* in *d* and *e*. Then bisect *a f* in *c*, and draw up the ridge Line of the Pediment *h c*; and from them the two gutter Lines *c e* and *c d*.

Lastly,

Lastly, Since the upper Part of the Building *p q t o*, stands in the same Plane with the Upright *b g l o*; thefore by the same Scale complete the Elevation thereof, and you will have completed the whole as required.

PROBLEM VIII.

There is the same Plans and Elevations given in an oblique View, to raise the Perspective Elevation thereof.

1. Let the Geometrical Plan be Plate 22. *Fig.* 3. and the Perspective Plan thereof *Fig.* 2. and let T be the Point of Sight.

2. Let T U be the ground Line of the Elevation *Fig.* 1, and A B the Horizon, and let B therein be the Point of Sight.

3. Transfer the hithermost Part of the Perspective Plan *Fig.* 2. to *Fig.* 1. making O P equal to C X, P Q equal to X Y, Q S equal to Y Z, and N S equal to U Z.

4. On the Angles U C X Y Z, raise the Perpendiculars U N, C O, X P, Y Q, and Z S.

4. Upon the Line O P, *Fig.* 1. raise and complete the Geometrical Elevation by the Scale of the Geometrical Plan M K L P O.

6. Upon the Angles N and S, raise the Perpendiculars N C and S F, making each equal to their respective Heights of 30 Feet, by the new proportion'd Scale as aforesaid, and then proceeding as in the former Problem, complete all the Windows, &c. therein.

7. Lay a Ruler from the Point of Sight B, to the Points I K M, and draw the ridge Line K G, terminating on the ridge Line D E, the Eve-lines M H and I L, terminating at H I. Then drawing the angle Line H Q you will have completed all the lower part of the Building.

8. Carry up the upper part *e f* H I, &c. as before directed, and from their Terminations draw Radials to the Point of Sight B, which will be intersected by the respective Perpendiculars L *g*, &c. of the Perspective Plan, and terminate their side Appearance as requir'd.

Note, The Hip C D, *Fig.* 1. is found by a Perpendicular rais'd from the Middle of the front End V W, at *x*, *Fig.* 2. over which the Angle of the Ridge D stands, and continu'd until it cut the Ridge Line D E in D; then drawing D C, it completes the Hip as required, and the like of the opposite Hip E F, standing over the Middle of H Z in *Fig.* 2.

PROBLEM IX.

The same Plan being given, with one of its concave Angles in Front, to raise the Perspective Elevation thereof as Plate 23. *Fig.* 1.

1. Let

1. Let the given Plan be *Fig.* 2. with the concave Angle *e k l*, plac'd in a direct View, let *n i* be the ground Line, and A *b* B, the Points of Distance and Sight.

2. Let G H be the ground Line of the Perspective Elevation, A C the Horizon thereof, and A B C the Points of Sight and Distance.

3. Now since G H *Fig.* 1. represents *n i Fig.* 2. therefore from the front Angles *e* and *l*, raise the Perpendiculars *e* E and *l* L, as also *k* K, *f* F, and *m* M.

4. Make E D and L N, *Fig.* 1. each equal unto the given Height of the Cornish, *viz.* 30 Feet, and draw the Lines D A, E A, cutting *f* F, being continu'd in T. Also from the Points N L draw the Lines N C and L C, cutting *m* M in M and O being continu'd; then will T D F E and N O L M be two Fronts of the Building, their Pediments T S D, and N O P only wanting.

5. From the Points *r* and *q*, over which the angular Points of the Pediment stands, raise the perpendicular Lines *r* R S and *q* Q P, and thro' the Points R Q where those Perpendiculars interfect the Line A E in R, and L C in Q, draw the Line *n* I.

6. Proportion the Scale of your Fronts on the Line *n* I, and by that new Scale set up the given Height of the Pediment O P and R S, and then drawing the ridge Lines of the two Pediments P A and S C, complete the Eve-lines P O, P N, and S T, S D. These ridge Lines of the Pediments are terminated first at X and Y by the perpendicular Lines *x* X, *x* Y, of the Perspective Plan rais'd from the Points *x x*, over which the Points X and Y do stand: And secondly, at *z* Z and *a*, by the Perpendiculars *a* Z and *b a*.

7. The Hip Lines *c z* and *a b*, are terminated at *c* and *b*, by the Perpendiculars *f c* and *d b*, intersecting the Line D C in *b*, and N A in *c*.

8. The Lines E K and K L being drawn to the Points of Distance A and C, as also the Lines N W and D W, and joining W K, you will complete all the lower part of the Building.

9. Since the upright Angle of the Tower *f* B is in the same Plane with the Angle W K, therefore through the Point K draw the Line F K M, and transfer the Scale of G H thereto.

10. Make W B equal unto the given Height (be it at Pleasure) by the last produc'd Scale of Feet of the Line F K M whereon it stands, and from the Point B draw the Lines B C, B A.

Lastly, From the respective Angles of the Plan *h* and *g* raise the Perpendiculars *h e* and *g d*, which will intersect B C in *d*, and B A in *e*.

In like manner set off the Heights, &c. of the Windows on W B, and terminate them by perpendicular Lines rais'd from the Plan, as the whole Body were, and you'll complete the Elevation as required.

Now from what I have here deliver'd, it is plain, that all the Angles of a Building are terminated by the respective Angles of its perspective Plan, and the Height of every Part is determin'd by the proportionable Scale of those Parts, whereby their Diminution is preserv'd, according to the Diminution of the Angles under which they are seen. This proportional Scale being duly consider'd, and the perspective Plans being truly executed, the Practice of Perspective will be as easy as delineating Geometrical Elevations.

N. B. The Views given in Plates 24, 25, 26, 27, 28, are added for the young Learner's Exercise, being the Consequences of the preceeding Problems.

F I N I S.

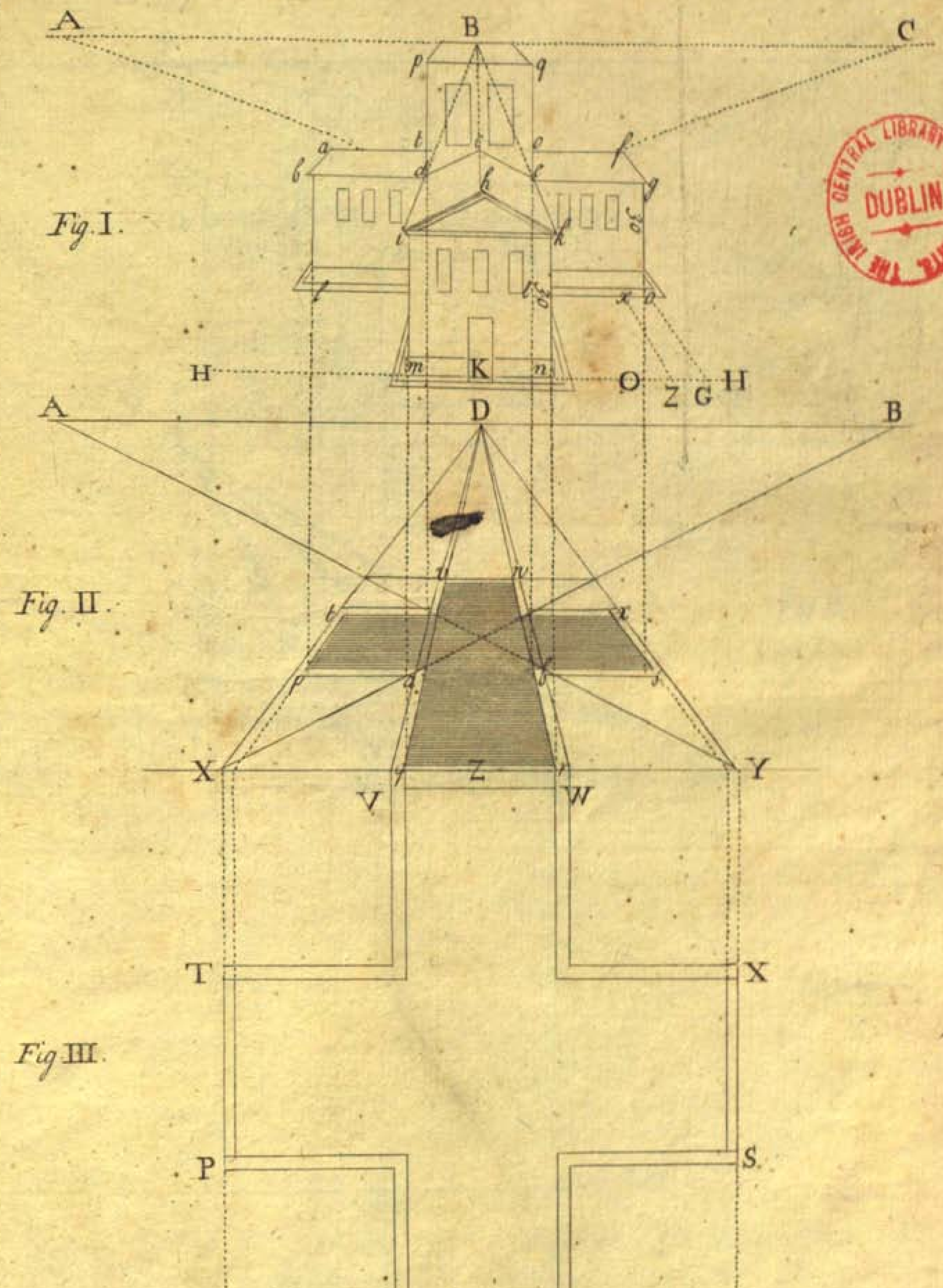

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Fig. I.
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Fig. II.
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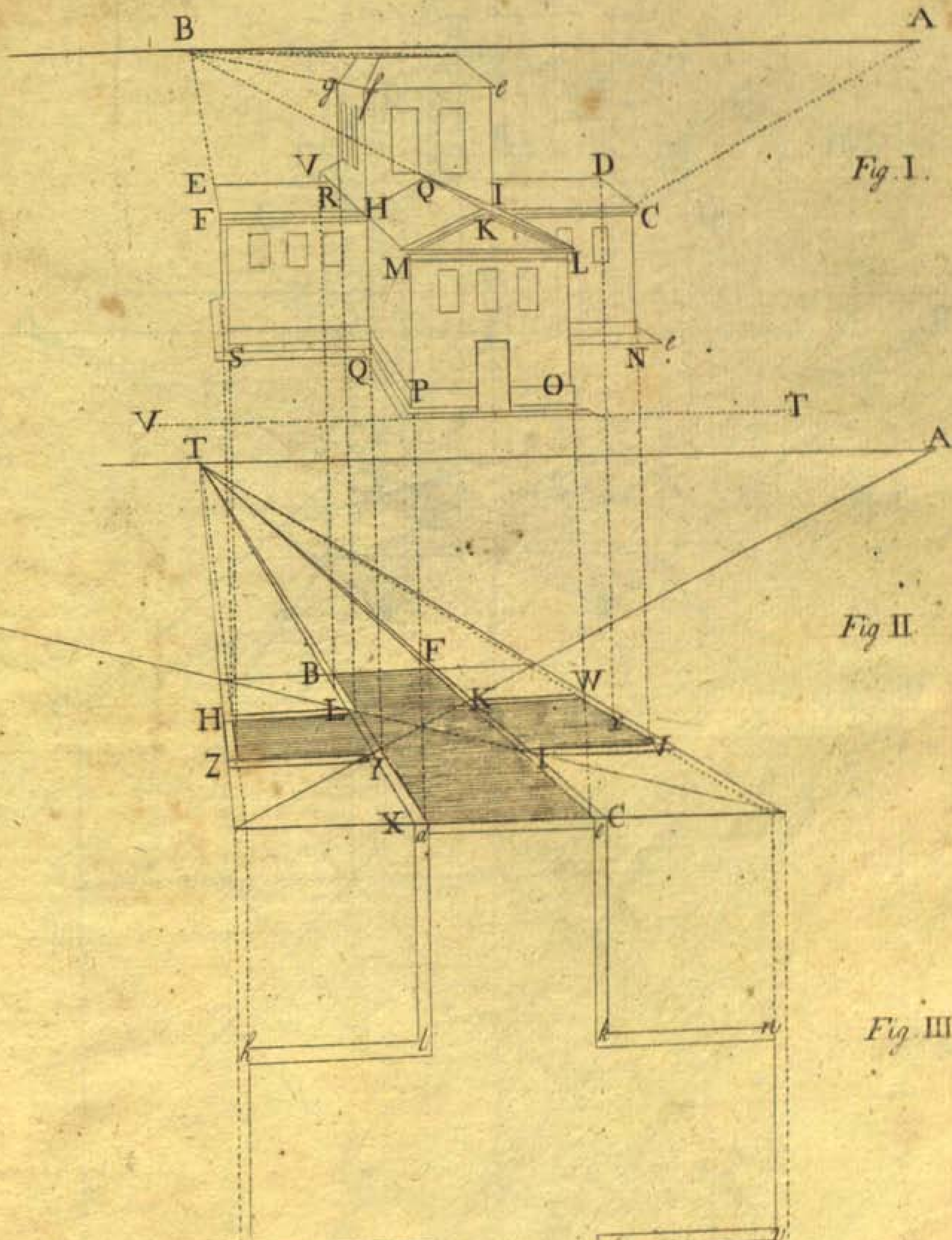
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Fig. I.
Fig. II.
Fig. III.

Fig I

Fig II

Fig III

Plate XXV

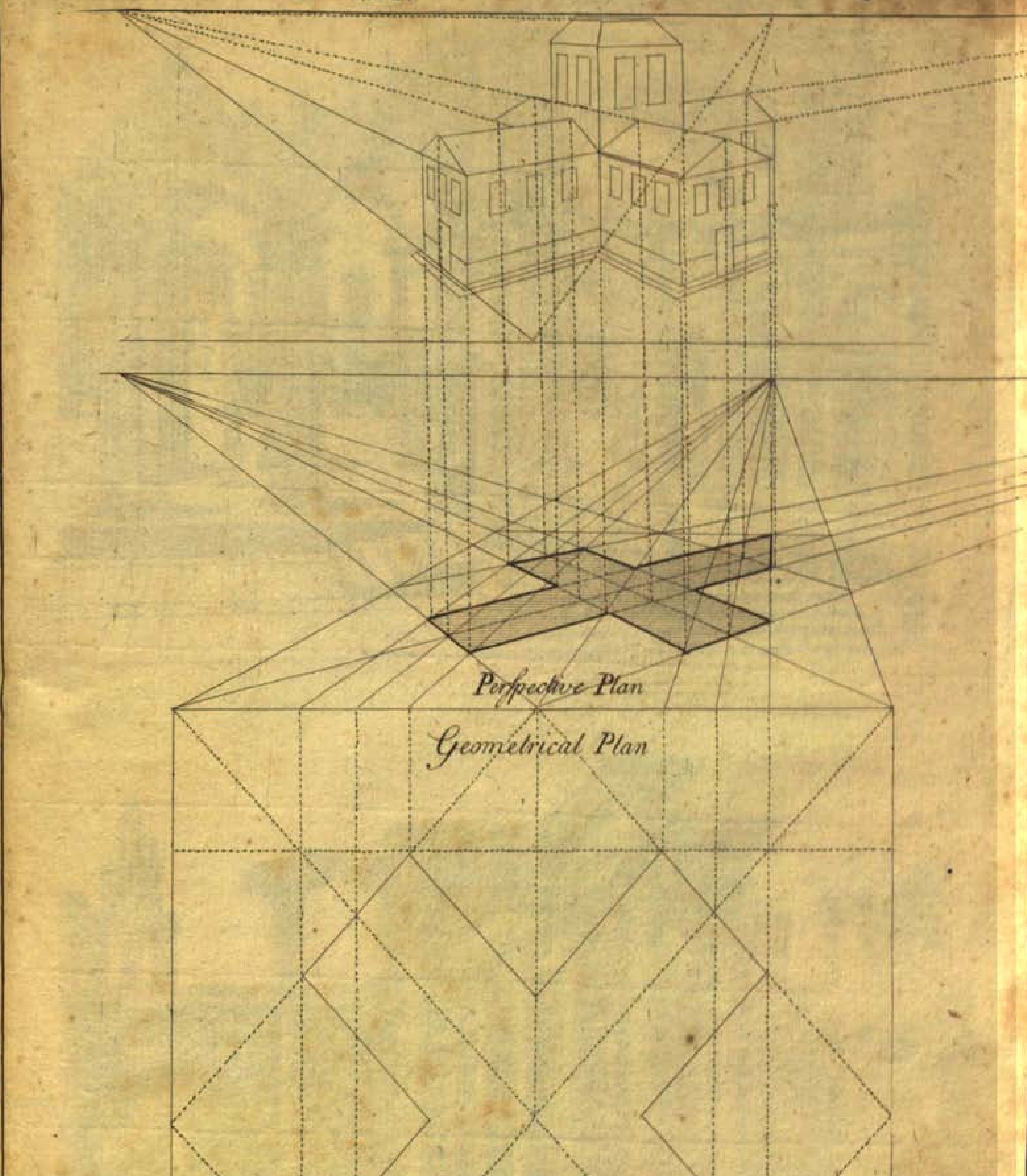

Plate XXV

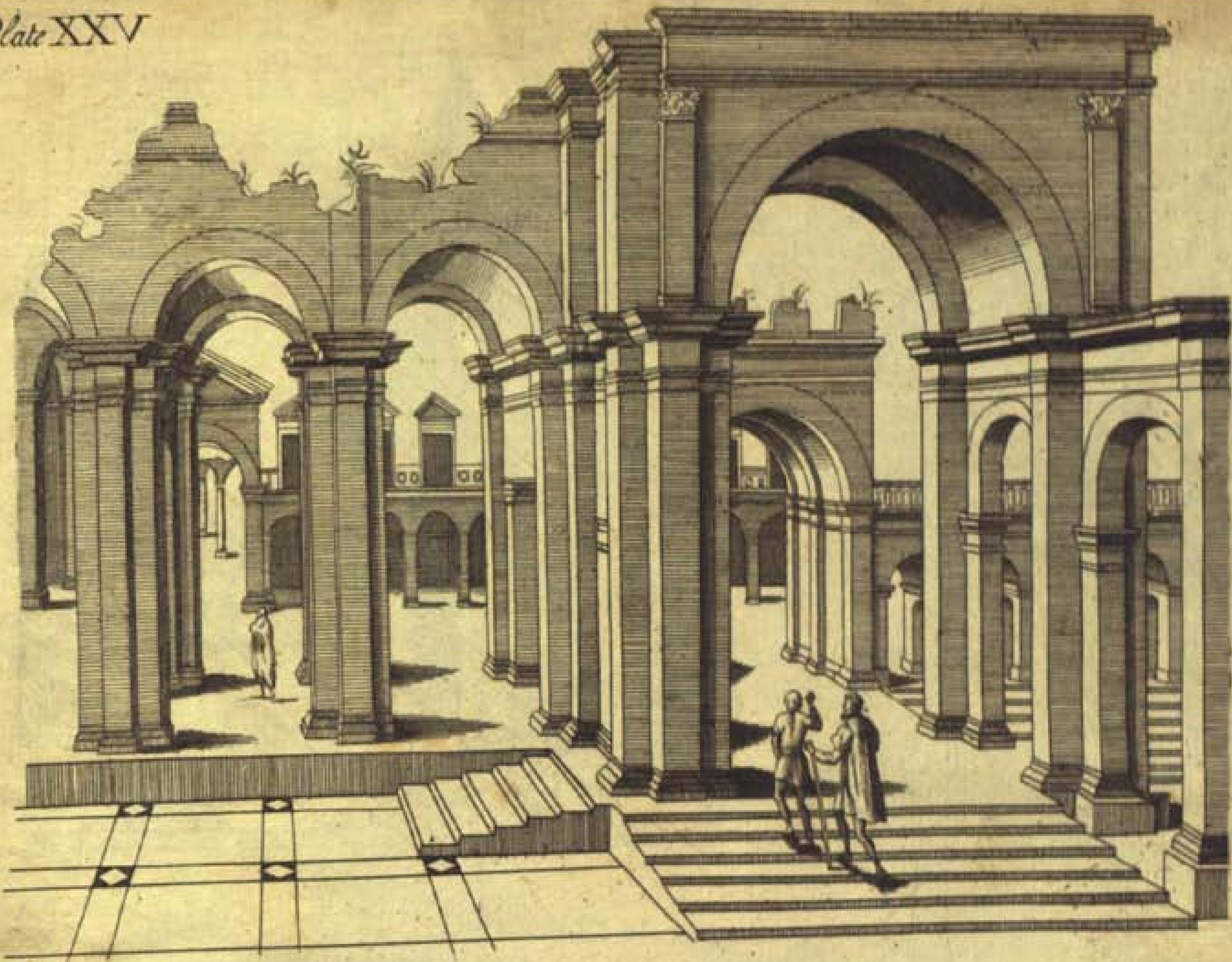

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Plate XXVI

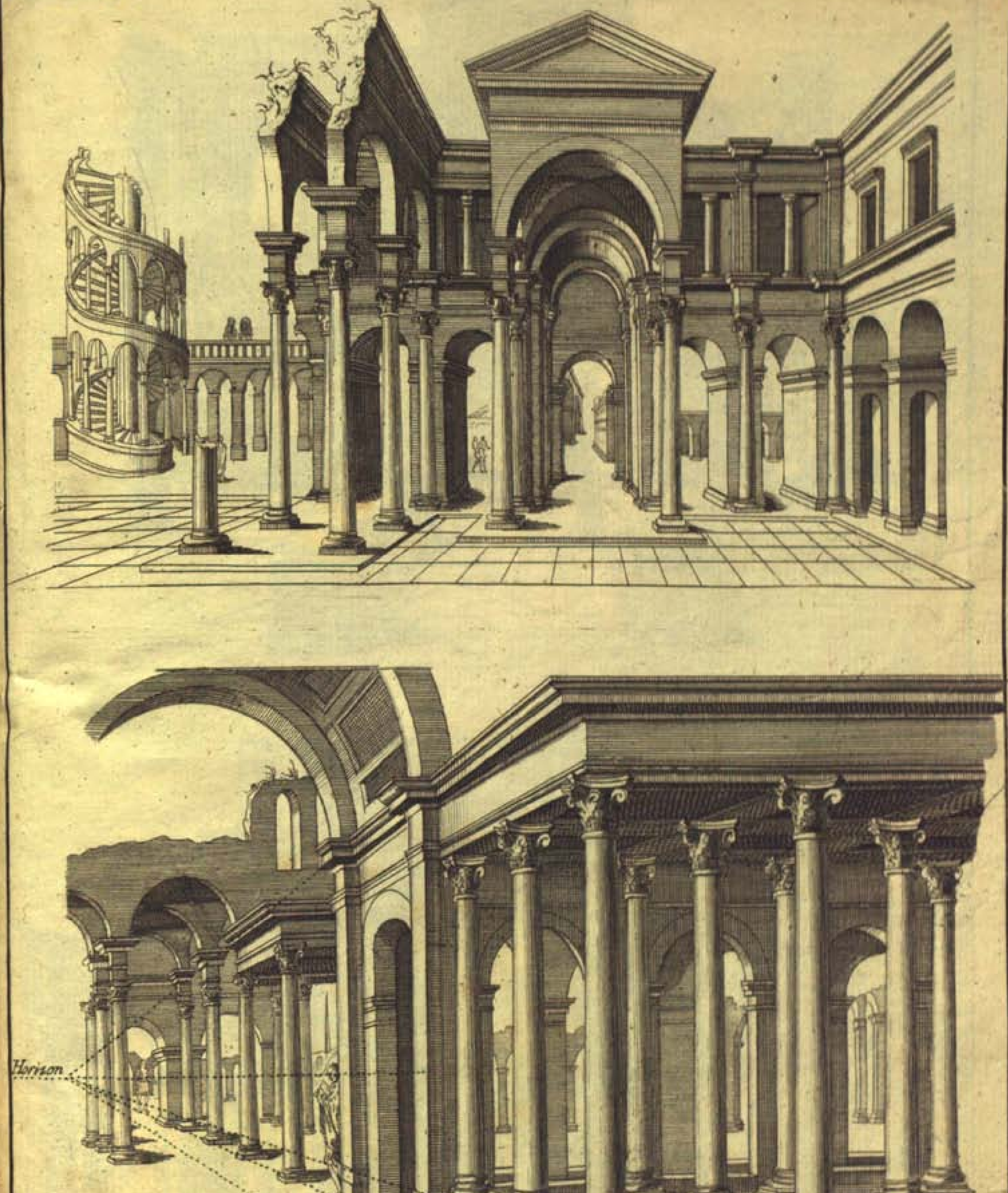

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Plate XXVII

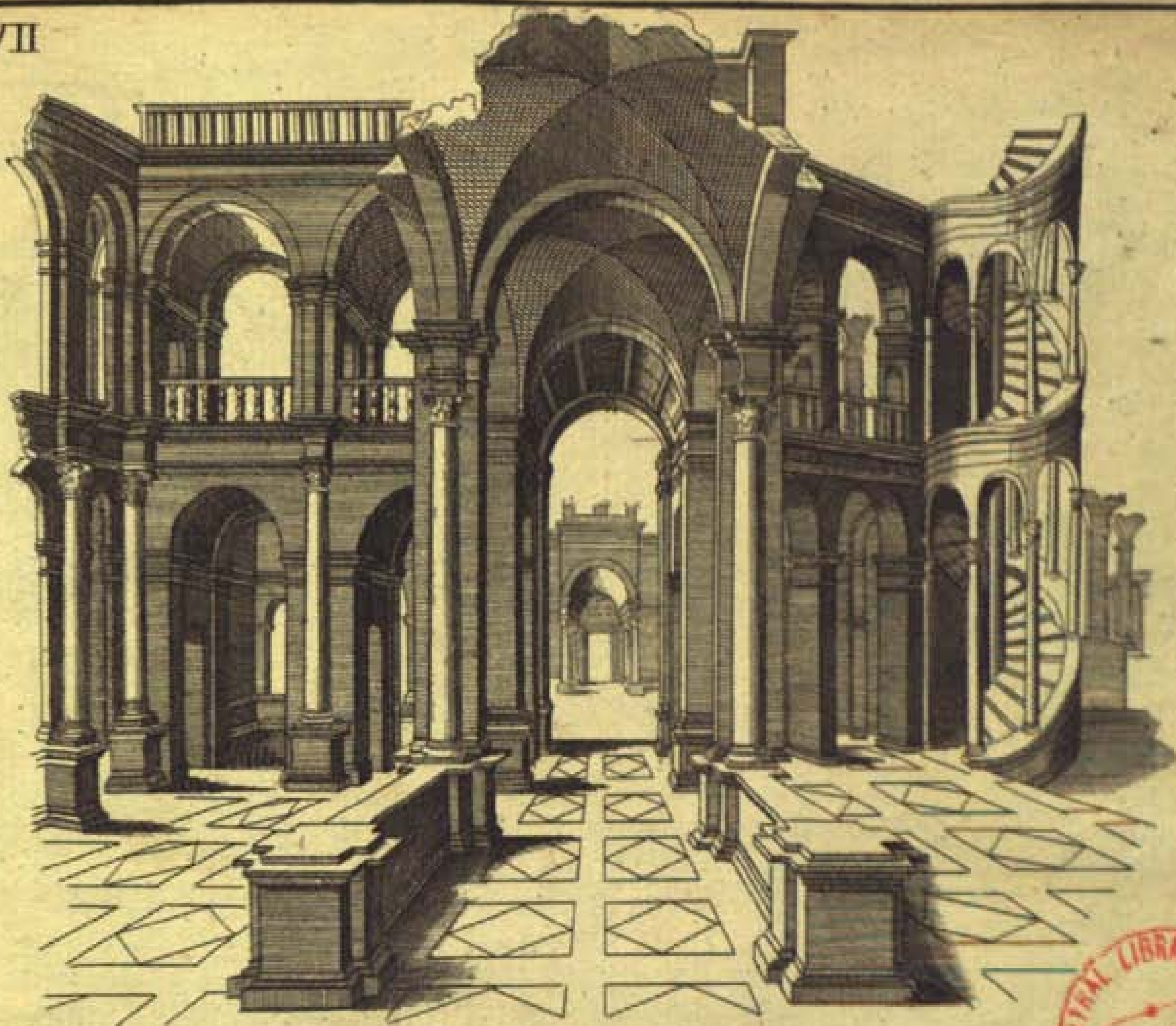

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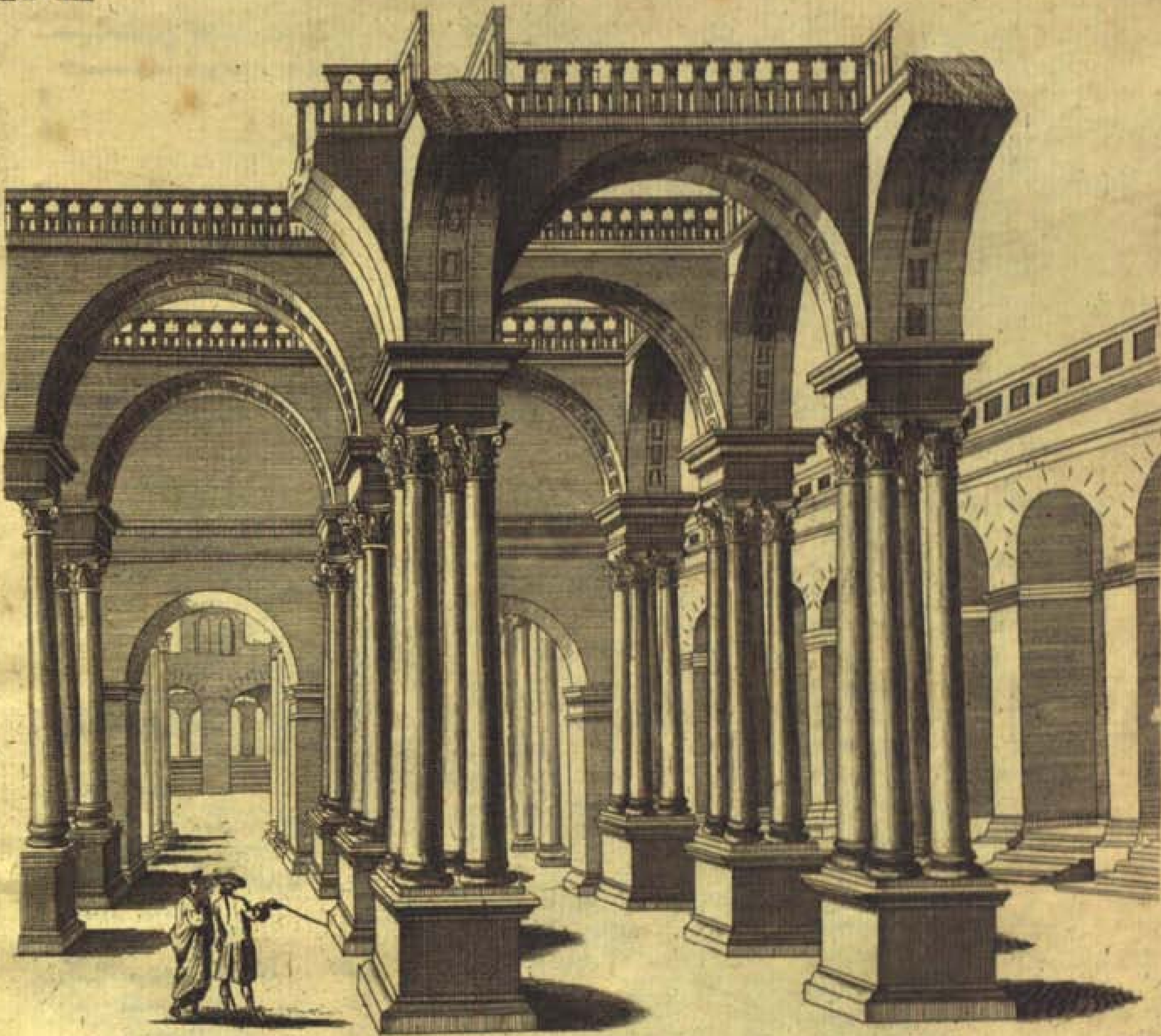
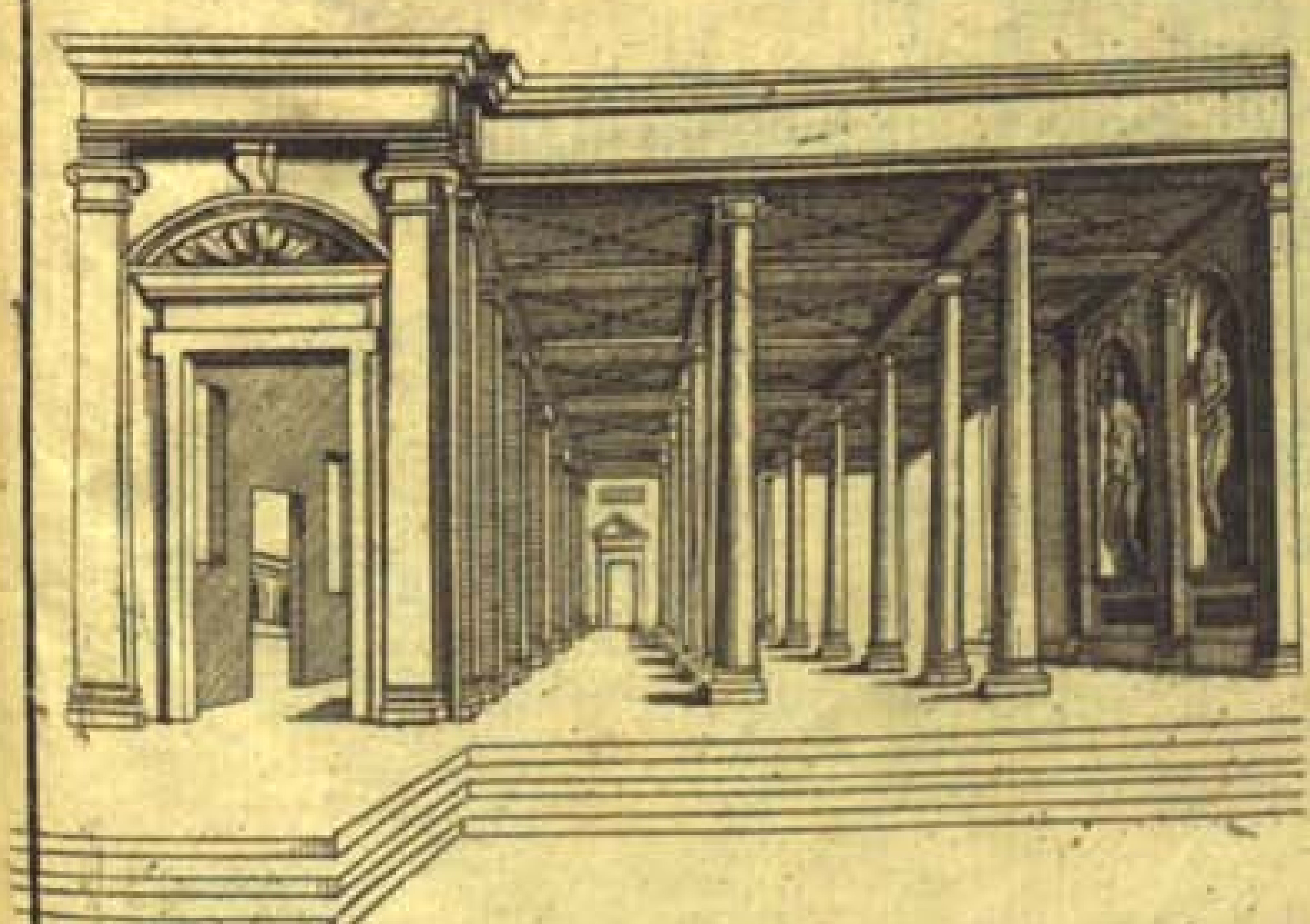
Plate XXVIII





B. Langley. delin

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